An - Najah Univ. J. Res.(N. Sc.) Vol. 30(1), 2016

On Unsteady MHD Flow Through Porous Medium Between Two Parallel Flat Plates

عن تدفق الماجنتوهايدروداينيمك المغناطيسية غير المستقرة من خلال وسط مسامي بين اثنين من اللوحات المسطحة المتوازية

AbdelLatif Sa'adAldin & Naji Qatanani
عبداللطيف سعدالدين، وناجي قطناني
Department of Mathematics, Faculty of Sciences, An-Najah National University, Nablus, Palestine
Corresponding Author, E-mail : abdel.latif@outlook.com
Received: (2/8/2015), Accepted: (21/12/2015)

Abstract

This paper deals with the unsteady Magnetohydrodynamics (MHD) flow of an electrically conducting, incompressible viscous fluid past through porous medium between two parallel plates in the presence of a transverse magnetic field and Hall effect. An exact solution based on Laplace transform has been presented. For the numerical simulation of the problem we have employed the finite difference scheme. The effects of M (Hartman number), m (Hall parameter) and K (Darcy parameter) on the primary velocity have been investigated and their profiles are shown graphically by using the Matlab software.

Keywords: Unsteady MHD Flow , Porous Medium , Hall current , Hartman number , Darcy parameter.

ملخص

في هذه الدراسة تم التعامل مع التدفق غير المستقر لماجنيتوهايدروداينمك المغناطيسية لسائل موصل كهرابائي غير قابل للانضغاط لزج من خلال وسط مسامي بين اثنين من اللوحات المسطحة المتوازية بوجود المجال المغناطيسي المستعرض وتأثير

تيار هول. لايجاد الحل النظري استعملنا تحويلات لابلاس واستخدمنا طريقة الفروق المتهية لايجاد الحلول العددية. لقد تم دراسة تاثير كل من عدد هارتمن و معلمة هول و معلمة دارسي على السرعه الرئيسية ، ولتمثيل النتائج و الحصول عليها استعملنا برنامج الماتلاب ، حيث تم عرض النتائج بيانيا.

الكلمات المفتاحية : التدفق غير المستقر لماجنيتوهايدروداينمك، وسط مسامي، تيار هول، عدد هارتمن، معلمة دارسي.

1 Introduction

The study of flow through porous media has been the object of scientific and engineering research in recent years. A porous medium is a material containing voids such as beach sand, sandstone, wood and the human lung. The concept of porous media which depends on Darcy's experimental law is used in many areas of applied science and engineering for example: filtration, soil mechanics and petroleum engineering.

Ram and Mishra (G. Ram & Mishra, 1977) investigated the unsteady flow through Magnetohydrodynamic porous media. Ram and Jain (P. C. Ram & Jain, 1990) have discussed MHD free convective flow through a porous medium in a rotating fluid. Reddy and Bathaiah (Reddy & Bathaiah, 1982) have analyzed the Hall effects on MHD flow through a porous straight channel. Islam and Biswas (Islam, Biswas, Islam, & Mohiuddin, 2011) have studied the MHD micropolar fluid flow through vertical porous medium. Qatanani, Barham and Musmar (Qatanani, Barham, & Musmar, 2012) have studied the analysis of aligned MHD plane flow in porous media in presence of magnetic field. Chauhan and Rastogi (Chauhan & Rastogi, 2012) have analyzed the Hall effects on MHD slip flow and heat transfer through a porous medium over an accelerated plate in a rotating system. Saha and Chakrabarti (Saha & Chakrabarti, 2013) have investigated the impact of magnetic field strength on magnetic fluid flow through a channel. Moniem and Hassanin (Moniem & Hassanin, 2013) have developed a solution of MHD flow past a vertical porous plate through a porous medium under oscillatory suction. Ahmed, Khan, Zaidi, Jan, Waheed and

Mohyud-Din (Ahmed et al., 2014) have discussed the MHD flow of an incompressible fluid through porous medium between dilating and squeezing permeable walls. Sa'adAldin and Qatanani (Sa'adAldin & Qatanani, 2015) have studied the unsteady MHD flow through two parallel porous flat plates. The effects of Hartman number and Hall parameter on the primary velocity have been investigated.

In this paper the effect of Hall current on the flow of an electrically conducting, incompressible viscous fluid past through porous medium between two parallel plates under a uniform transverse magnetic field is considered. An exact solution based on Laplace transform method has been presented. For the numerical simulation of the problem we have employed the finite difference scheme. Numerical results have shown to be in a good agreement with the exact ones.The effects of M (Hartman number), m (Hall parameter) and K (Darcy parameter) on the primary velocity have been investigated and their profiles are shown graphically.

2 Formulation of The Problem

We consider an unsteady flow of an electrically conducting, incompressible viscous fluid past through porous medium between two parallel plates with Hall effect. Let the x-axis be taken along the plates and y-axis normal to the plates. The fluid is subjected to a constant transverse magnetic field of strength B_0 in the y direction, where the flow is considered in the x direction, as illustrated in Figure 2.1 . The governing equations for the unsteady, viscous incompressible flow of an electrically conducting fluid for the Brinkman-extended Darcy model are:

Equation of continuity :

$$\nabla.\mathbf{q} = 0 \tag{2.1}$$

Equation of motion :

$$\frac{\partial \mathbf{q}}{\partial t} + (\mathbf{q}.\nabla)\mathbf{q} = -\frac{1}{\rho}\nabla \mathbf{p} + \frac{\mu}{\rho}\nabla^2 \mathbf{q} - \frac{\mu}{\rho}\frac{\mathbf{q}}{k} + \frac{1}{\rho}\mathbf{J}\times\mathbf{B} \tag{2.2}$$

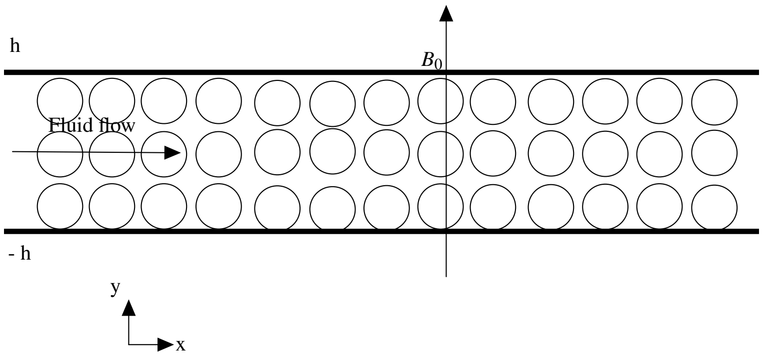

Figure (2.1): Schematic diagram of the system.

General Ohm's law :

$$\mathbf{J} + \frac{\omega\tau}{B_0} J \times \mathbf{B} = \sigma[E + \mathbf{q} \times \mathbf{B} + \frac{1}{\rho_e n_e} \nabla \mathbf{P}_e] \tag{2.3}$$

Gauss's law of magnetism :

$$\nabla . \mathbf{B} = 0 \tag{2.4}$$

where q is the velocity vector, ρ is the fluid density, p is the pressure, J is the current density, B is the magnetic vector, μ is the co-efficient of viscosity, σ the electrical conductivity, k is the permeability of the medium, ω is the electron frequency, τ is the electron collision time, ρ_e the electric charge, n_e is the number density of electron, P_e is the electron pressure and E is the electric field.

We assume E to be negligible and the magnetic Reynold's number is small so that magnetic induction effect is ignored. Moreover, in the absence of pressure gradient, the ion-slip effects and electron pressure gradient we have

$$\mathbf{J} = (j_x, j_y, j_z) \quad \mathbf{q} = (u, 0, 0) \quad \mathbf{B} = (0, B_0, 0)$$

$$\mathbf{J} = \sigma \mathbf{q} \times \mathbf{B} - \frac{m}{B_0} \mathbf{J} \times \mathbf{B} \tag{2.5}$$

$$j_x = m j_z \tag{2.6}$$

$$j_y = 0 \tag{2.7}$$

$$j_z = \sigma B_0 u - m j_x \tag{2.8}$$

Solving (2.6) and (2.8) we have

$$j_x = \frac{\sigma B_0 m u}{(1+m^2)} \tag{2.9}$$

$$j_z = \frac{\sigma B_0 u}{(1+m^2)} \tag{2.10}$$

$$\frac{1}{\rho} \mathbf{J} \times \mathbf{B} = -\frac{1}{\rho} j_z B_0.$$

As the plates are infinite, there is no x dependance. Consequently equations (2.2) and (2.3) yield the following equations:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho (1+m^2)} - \nu \frac{u}{k} \tag{2.11}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y} \tag{2.12}$$

where u is the axial velocity , ν is the kinematic viscosity and $m = \omega\tau$ is the Hall parameter. The initial and boundary conditions are given by :

$$\begin{cases} u = 0, & t \leq 0 \\ u = 0, y = \pm h, & t > 0 \end{cases} . \tag{2.13}$$

Introducing the non-dimensional quantities :

$$Y = \frac{y}{h}, \quad T = \frac{\nu t}{h^2}, \quad U = \frac{u}{V}, \quad M^2 = \frac{\sigma B_0^2 h^2}{\mu}, \quad K = \frac{k}{h^2}, \quad P = \frac{ph}{\mu V}, \quad X = \frac{x}{h}$$

where M is the Hartman number, K is the Darcy parameter and V is the mean velocity of the fluid.

Hence the partial differential equations (2.11) and (2.12) with the initial and boundary (2.13) conditions become :

$$\frac{\partial U}{\partial T} = -\frac{\partial P}{\partial X} + \frac{\partial^2 U}{\partial Y^2} - (\frac{M^2}{(1+m^2)} + \frac{1}{K})U \tag{2.14}$$

$$0 = \frac{\partial P}{\partial Y} \tag{2.15}$$

subject to the initial and boundary conditions :

$$\begin{cases} U = 0, & T \leq 0 \\ U = 0, Y = \pm 1, & T > 0 \end{cases} . \tag{2.16}$$

3 Analytical Solution

Here we find an exact solution based on Laplace transform to the MHD flow problem presented in section two.

From equation (2.15) we see that the pressure is independent of Y. Then it is a function of T only. In this case we can take the pressure gradient as a constant quantity that is

$$\frac{\partial P}{\partial X} = -P_0$$

where $P_0 > 0$, thus equation (2.14) becomes

$$\frac{\partial U}{\partial T} = P_0 + \frac{\partial^2 U}{\partial Y^2} - (\frac{M^2}{(1+m^2)} + \frac{1}{K})U \tag{3.1}$$

Taking the Laplace transform of equation (3.1) with respect to the variable T we obtain :

$$\frac{d^2}{dY^2}\widehat{U}(s,Y) - (\frac{M^2}{(1+m^2)} + \frac{1}{K} + s)\widehat{U}(s,Y) = -\frac{P_0}{s} \tag{3.2}$$

where

$$\widehat{U}(s,Y) = L[U(T,Y)]$$

and

$$\widehat{U}(s,-1) = 0, \widehat{U}(s,1) = 0 \tag{3.3}$$

Solving equation (3.2) subject to (3.3) we get

$$\widehat{U}(s,Y) = -\frac{P_0 \cosh\sqrt{(\frac{M^2}{(1+m^2)} + \frac{1}{K} + s)}Y}{s(\frac{M^2}{(1+m^2)} + \frac{1}{K} + s)\cosh\sqrt{(\frac{M^2}{(1+m^2)} + \frac{1}{K} + s)}} + \frac{P_0}{s(\frac{M^2}{(1+m^2)} + \frac{1}{K} + s)} \tag{3.4}$$

Finding the Laplace inverse transform of equation (3.4) using the complex inversion formula we obtain

$$U(T,Y) = \frac{P_0}{(\frac{M^2}{(1+m^2)} + \frac{1}{K})}\left(1 - \frac{\cosh\sqrt{(\frac{M^2}{(1+m^2)} + \frac{1}{K})}Y}{\cosh\sqrt{(\frac{M^2}{(1+m^2)} + \frac{1}{K})}}\right) - \frac{16P_0}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n e^{s_n T}\cos((\frac{2n+1}{2})\pi Y)}{(\frac{M^2}{(1+m^2)} + \frac{1}{K} + \frac{(2n+1)^2\pi^2}{4})(2n+1)} \quad (3.5)$$

where

$$s_n = -\left(\frac{M^2}{(1+m^2)} + \frac{1}{K} + \frac{(2n+1)^2\pi^2}{4}\right), n = 0,1,2,....$$

4 Numerical Simulation and Discussion

The numerical handling of equation (3.1) subject to (2.16) can be carried out by using the finite difference scheme. The computational domain is divided into a mesh of lines parallel to Y and T axes. The implicit finite difference approximations for derivatives is given by:

$$\frac{\partial U}{\partial T}(Y_i,T_j) = \frac{U(Y_i,T_j) - U(Y_i,T_{j-1})}{k} - \frac{k}{2}\frac{\partial^2 U}{\partial T^2}(Y_i,T_j^*) \quad (4.1)$$

where $T_j^* \in (T_{j-1},T_j)$.

$$\frac{\partial^2 U}{\partial Y^2}U(Y_i,T_j) = \frac{U(Y_{i+1},T_j) - 2U(Y_i,T_j) + U(Y_{i-1},T_j)}{h^2} - \frac{h^2}{12}\frac{\partial^4 U}{\partial Y^4}U(Y_i^*,T_j) \quad (4.2)$$

where $Y_i^* \in (Y_{i-1},Y_{i+1})$.

Substituting (4.1) and (4.2) into the corresponding partial differential equation (3.1) we obtain an appropriate set of finite difference equations

$$\frac{w_{i,j}-w_{i,j-1}}{k}=P_0+\frac{w_{i+1,j}-2w_{i,j}+w_{i-1,j}}{h^2}-\Big(\frac{M^2}{(1+m^2)}+\frac{1}{K}\Big)w_{i,j} \quad (4.3)$$

where $w_{i,j}$ approximates $U(T_i,Y_j)$. Solving equation (4.3) for $w_{i,j-1}$ we obtain

$$w_{i,j-1}=-\lambda w_{i+1,j}+(1+2\lambda+\Big(\frac{M^2}{(1+m^2)}+\frac{1}{K}\Big)k)w_{i,j}-\lambda w_{i-1,j}-kP_0 \quad (4.4)$$

where $\lambda=\frac{k}{h^2}$, k and h are mesh sizes along time T and Y directions respectively. Putting this into matrix form yields

$$\begin{bmatrix} 1+ak+2\lambda & -\lambda & 0 & \dots & 0 \\ -\lambda & \ddots & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & -\lambda \\ 0 & \ddots & 0 & -\lambda & 1+ak+2\lambda \end{bmatrix} \begin{bmatrix} w_{i,j} \\ \vdots \\ \vdots \\ \vdots \\ w_{m-1,j} \end{bmatrix} = \begin{bmatrix} w_{1,j-1}+kP_0 \\ \vdots \\ \vdots \\ \vdots \\ w_{m-1,j-1}+kP_0 \end{bmatrix}$$

where

$$a=\Big(\frac{M^2}{(1+m^2)}+\frac{1}{K}\Big)$$

To draw a comparison between the analytical and the numerical solutions presented in sections 2 and 3 respectively, we consider the following test case with $M=1$, $m=1$, $K=0.1$, $P_0=1$, $T=0.25$ fixd and $Y\in[0,1]$.

Table 4.1 compares both the exact and the numerical values for the primary velocity $U(Y,T)$. A further comparison between the exact and numerical values for the primary velocity can be observed in Figure 4.1. A

Table (4.1): The exact and numerical solutions of the primary velocity U.

Y	Exact solution U_E	Approxmiate solution U_n	Error $=\|U_E - U_n\|$
-0.8	0.0439256214739	0.0439284690437	$0.2847569757748 \times 10^{-5}$
-0.6	0.0662966816784	0.0662354201183	$0.6126156005674 \times 10^{-4}$
-0.4	0.0773731593814	0.0772434172904	$0.1297420909940 \times 10^{-3}$
-0.2	0.0824053505275	0.0822283020686	$0.1770484588176 \times 10^{-3}$
0	0.0838631333124	0.0836692897549	$0.1938435575730 \times 10^{-3}$
0.2	0.0824792484052	0.0823013886500	$0.1778597551711 \times 10^{-3}$
0.4	0.0775578709141	0.0774266269162	$0.1312439979023 \times 10^{-3}$
0.6	0.0666815959272	0.0666185173810	$0.6307854626647 \times 10^{-4}$
0.8	0.0446904326399	0.0446921159260	$0.1683286084617 \times 10^{-5}$

plot of the absolute error resulted from the approximation can be seen in Figure 4.2.

Further investigation for the numerical calculation of the primary velocity U for different values of Hartman number M, Hall parameter m, Darcy parameter K and normal coordinate Y keeping the value of time T fixed at $T = 0.5$ have been carried out.

The effect of Hartman number M on the variation of the primary velocity U can be seen in Figuer 4.3, while Figure 4.4 illustrates the effect of the Hall parameter m on the fluid velocity with the assigned values of $M = 1$ and $K = 0.1$.

Finally, Figure 4.5 shows the effect of Darcy parameter K on the velocity of the flow with $M = 1$ and $m = 1$.

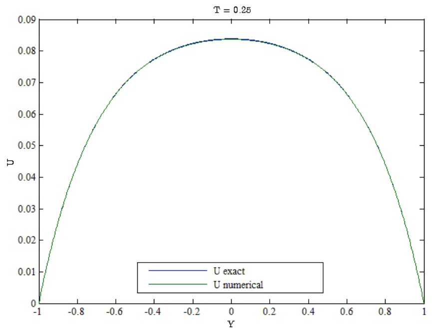

Figure (4.1): The exact and numerical values for the primary velocity.

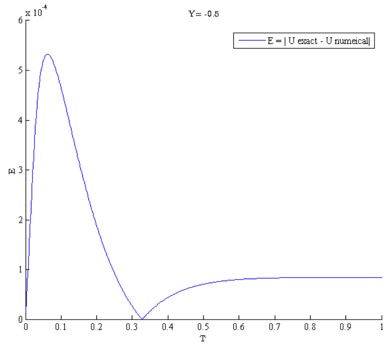

Figure (4.2): The absolute error resulted from the approximation.

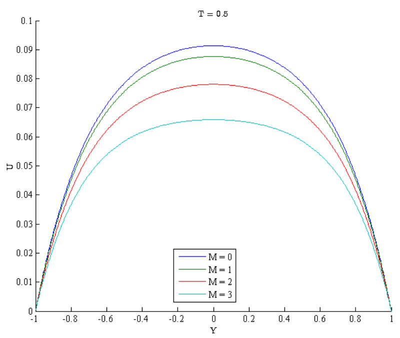

Figure (4.3): Primary velocity profiles with several values of M using $m = 1$ and $K = 0.1$.

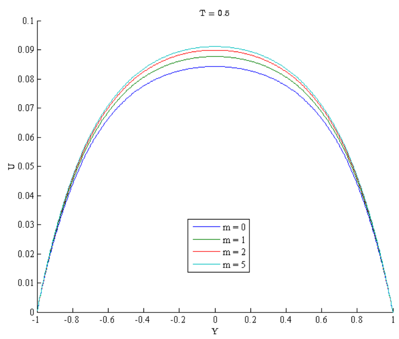

Figure (4.4): Primary velocity profiles with several values of m using $M = 1$ and$K = 0.1$.

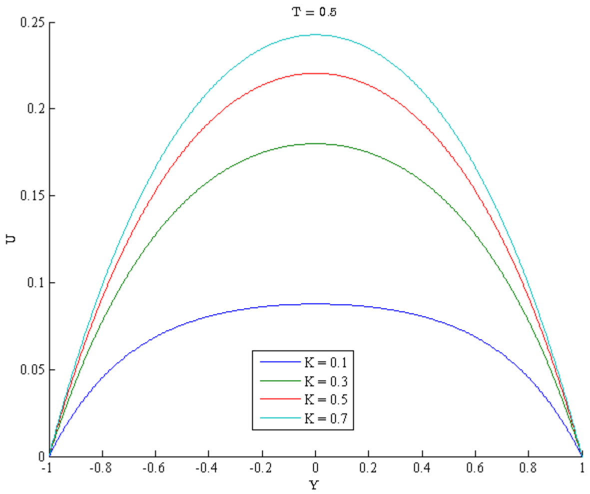

Figure (4.5): Primary velocity profiles with several values of K using $M = 1$ and $m = 1$.

5 Conclusions

In this work, the problem of unsteady MHD flow through porous medium between two parallel flat plates has been investigated and solved using the Laplace transform method and the finite difference technique. The exact and numerical results have shown to be in a closed agreement. This can clearly be seen in Figures 4.1 and 4.2. The magnetic field parameter M slows down the velocity of the flow field at all points due to the magnetic pull of the Lorentz force acting on the flow field. This can be noticed in Figure 4.3. Moreover, in Figure 4.4, we see that the primary velocity is enhanced by increasing the permeability K of the porous medium because the Darcian resistance to fluid flow through a porous medium is inversely proportional to K. In fact these observations agree with (G. Ram & Mishra, 1977). Figure 4.5 represents the effect of Hall parameter m on the flow through porous medium. It is clear that due to the increase in the Hall parameter m value there is a rise in the primary velocity.

References

- Ahmed, N. Khan, U. Zaidi, Z. A. Jan, S. U. Waheed, A. & Mohyud-Din, S. T. (2014). *MHD flow of incompressible fluid through porous medium between dilating and squeezing permeable walls*. Journal of porous media, 17(10), 861-867.

- Chauhan, D. S. & Rastogi, P. (2012). *Hall effects on MHD slip flow and heat transfer through a porous medium over an accelerated plate in a rotating system*. International journal of nonlinear science, 14(2), 228-236.

- Islam, A. Biswas, H. A. Islam, R. & Mohiuddin, S. M. (2011, November). *MHD micropolar fluid flow through vertical porous medium*. Academic research international, 1(3), 381-393.

- Moniem, A. A. & Hassanin, W. S. (2013). *Solution of MHD flow past a vertical porous plate through a porous medium under oscillatory suction*. Applied mathematics, 4, 694-702.

- Qatanani, N. Barham, A. & Musmar, M. (2012, June). *On the analysis of aligned MHD plane flow in porous media in presence of magnetic field*. International journal of modern engineering research (IJMER), 2(3), 1055-1061.

- Ram, G. & Mishra, R. S. (1977). *Unsteady flow through magnetohydrodynamic porous media*. Indian J. pure appl. math., 8, 637-647.

- Ram, P. C. & Jain, R. K. (1990). *MHD free convective flow through a porous medium in a rotating fluid*. International journal of energy research, 14(9), 933-939.

- Reddy, N. B. & Bathaiah, D. (1982, October). *Hall effect on MHD flow through a porous straight channel*. Defence science journal, 32(4), 313-326.

- Sa'adAldin, A. & Qatanani, N. (2015, June). *Analytical and numerical methods for solving unsteady MHD flow problem*. International J. of Math. Sci. and Engg. Appls. (IJMSEP), 9(2), 307-318.
- Saha, S. & Chakrabarti, S. (2013, July). *Impact of magnetic field strength on magnetic fluid flow through a channel*. International journal of engineering research and technology (IJERT), 2(7), 1-8.