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Simplified Equations to Find the Relationship Between Soil and Structure Displacements for Rectangular Single Column and Footing on Dry Voidless Cohesionless Soil Due to Vertical Loads

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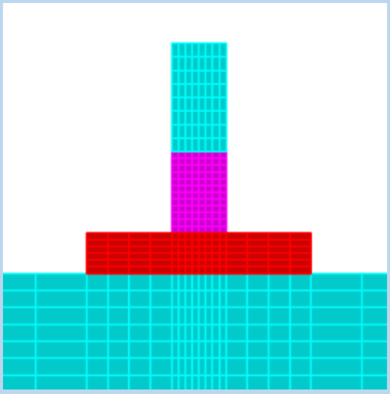
Abstract: Usually, structural designers are assuming the soil as rigid object when they design small to intermediate structures, or assuming the settlement of soil to be always within the acceptable range. However, the engineering practice shows that high percentage of cracks and failures in structures are related to soil settlement reasons, non-uniform soil settlement for example. Geotechnical engineers developed many methods to calculate the settlement of soil, but the majority of these methods are geotechnical engineering oriented, and standard structural engineer may struggle using them. Therefore, because of the importance of finding the approximate soil settlement in many cases during the design process or during the implementation of the structure, this study gives simple equations that can predict the soil settlement with acceptable accuracy, by creating a relationship between the soil settlement and the displacement of the structure. The targeted equation will allow us to find the soil settlement from knowing the approximate displacement of the structure. Such an equation gives a conceptual value to be used in 3D analysis of structures using computer programs like SAP2000 and ETABS. The targeted structure in this study is a simple one rectangular column and footing structure.



Keywords: Soil-structure interaction, direct method, modulus of elasticity, rigidity, soil settlement, displacement of soil, dry soil, sha.

Introduction

Two main assumptions are controlling the engineering practice regarding soil - structure interaction. Structural engineering have the assumption of flexible-structure rigid-soil (Lai and Martinelli, 2013), while the geotechnical engineering have the assumption of flexible-soil rigid- structure (Poulos and Hull, 1989).

Although these assumptions are opposite, they are widely used by both geotechnical and structural engineers to facilitate the design and simplify the calculations. However, these assumptions do not represent reality, where both the structure and the soil are flexible within one soil-structure interaction system (Lai and Martinelli, 2013).

Soil-structure interaction can be defined as “an interdisciplinary field of endeavor which lies at the intersection of soil and structural mechanics for both static and dynamic behaviors” (Kausel, 2010).

Geotechnical engineers developed many methods to find the soil settlement (Das, 2009). However, these methods are oriented for geotechnical engineering uses, which are sometimes difficult or inaccurate for an average structural engineer to use. Other theoretical methods (Holtz, 1991) considered both soil and structure as flexible objects for a certain limit, which this study will conduct a comparison between the theoretical results and the FEM’s results.

Therefore, the goal of this research is to find simple equations to calculate the soil immediate settlement, for cohesionless soil, easily with acceptable accuracy using the displacement of the structure as a reference, where the displacement of the structure can be found during the structural design process. The equations will benefit the engineer by expecting the soil settlements that might occurred due to known stress, or understand an already occurred soil settlement due to situated structure.

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Methodology

The purpose of the paper is to study the relationship between the displacement of structure and the settlement of soil, in order to obtain equations to find the settlement of soil when the displacement of structure is known. The displacement of structure depends on the stress affecting the structure, either service or ultimate, depending on the type of the analysis needed. For this paper, the service load will be used.

The soil settlement under the center of the footing will be recorded, in addition to the displacement of structure, which includes the displacement of the column and the footing.

A comparison will be conducted between the FEM's results and the results calculated from the analytical method, to note the differences occurred due to the change of soil rigidity on the results of the equations.

In order to simplify the procedure, the results will be presented as ratios to the total displacement; the ratio of soil settlement to total displacement $\frac{\Delta_{soil}}{\Delta_{total}}$ and the ratio of displacement of structure to total displacement $\frac{\Delta_{structure}}{\Delta_{total}}$.

The results will be presented as curves, studying the effect of each variable on the overall behavior of the system, which will be taken into consideration when concluding the equations.

The displacement ratios are dependent, therefore by knowing the displacement ratio of the structure the displacement ratio of soil can be found. From the ratios, in addition to the displacement of structure alongside the dimensions of column, footing and the modulus of elasticity of the soil and structure, the soil settlement can be found. (Touqan et all, 2017).

Numerical model description using SAP2000

Soil structure interaction analysis method

Engineers developed many methods in order to deal with the soil-structure interaction analysis. There are the direct and the indirect approaches. According to (Lai and Martinelli, 2013), in the direct approach the soil volume and the structure are both part of the same model which is analyzed in a single step by using one of several numerical discretization techniques (e.g. Finite Element Method, Spectral Element Method, Finite Difference Method, etc.).

This study will use the direct approach. To achieve the purpose of the study, a detailed three-dimensional multi noded structural model is created using the finite element program SAP2000 (CSI, 2017).

Geometric properties

This paper study a simple single rectangular footing with a rectangular column, having a fixed height of 3m, which is the standard length used in many parts of the Middle East. As seen in Figure (**1** and Figure (**2** The footing has two dimensions of $l1, l2$, with depth d, while the column has dimensions of $c1, c2$, with stress appointed on the top face of the column. The soil was represented with sufficient dimensions to surpass the effect of the stress curves presented in (Das,2008).

The mesh used in the finite element program SAP 2000 was designed in order to have an acceptable accuracy with minimal calculation time. The soil has two main mesh systems: fine mesh near the structure, and more course one far from the stress curves. Similarly, the structure has finer mesh at the joint volume between the column and the footing, and the mesh becomes courser when going up through the column's height, as shown in Figure (**3**.

Material properties

Soil and structure are assumed fully elastic, homogeneous and isotropic in order to simplify the model and the calculations (Kocak and Mengi, 2000). The soil is assumed dry, cohesionless with no water pores to find the immediate settlement only while ignoring the consolidation settlement (Bowles, 1982). The modulus of elasticity of soil can be used as a main property and parameter for the calculations of settlements (Poulos and Hull ,1989), (Das, 2009) and (Holtz and Kovaks ,1981) who stated that "the immediate, or distortion, settlement, although not actually elastic is usually estimated by using elastic theory".

The soil modulus of elasticity vary from a very soft soil (5MPa), to a very rigid soil (500000MPa) (Geotesting.info, 2020). The modulus of elasticity of soil can approach approximately 15000 MPa for limestone for example (Abdulhadi and Barghouthi, 2012), but higher modulus of elasticity that exceed this value is used to obtain the upper limit of the modulus of elasticity ratio.

The structural material of the column and footing is concrete, and it is assumed a homogenous, isotropic and elastic material, with modulus of elasticity of 24500 MPa. The densities of both soil and structure are neglected for reasons will be explained later.

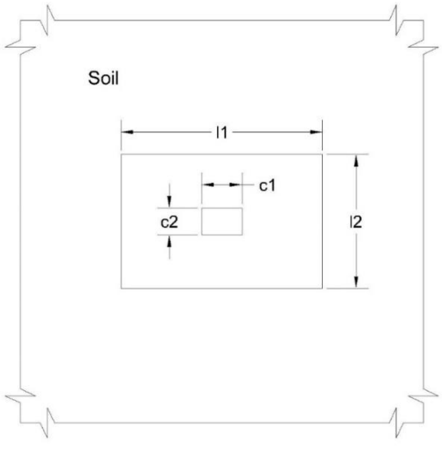

Figure (1): Top view of the soil-structure interaction system.

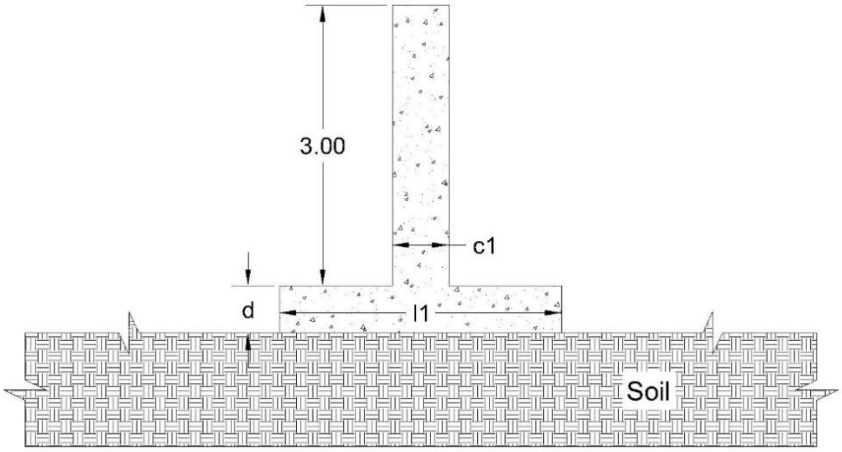

Figure (2): Side view of the soil-structure system.

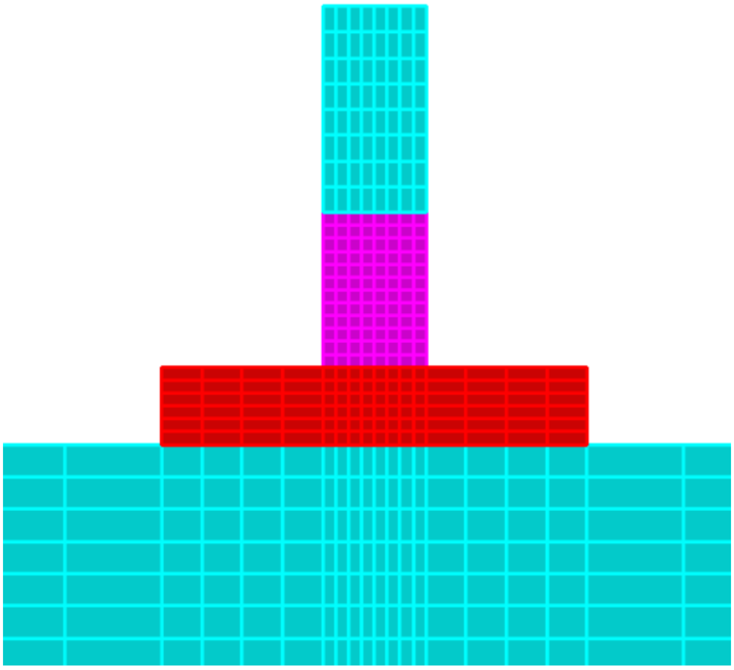

Figure (3): Side view from the meshed soil-structure system used in SAP 2000.

Basic assumptions

The structure is a simple structure of on rectangular column and footing placed on soil, The height of the column is considered fixed with height of 3m. Poissons ratio (v) is also considered constant with value of 0.2; which is value found to be approximately constant due to compression loads for concrete within the elastic limits (Kupfer et al. 1969).

Poissons ratios for soil are between the range 0.1 – 0.4, noticing that all soil types are sharing the ratio 0.3 (Kulhawy and Mayne,1990) which is the value used in the models.

As the footing is shallow, the soil weight over the footing is neglected, because the soil stress affecting footing is very low in shallow footings when compared with the active stress from the structure.

The main variables are the dimensions of the column and footing, in addition to the modulus of elasticity of the concrete and the soil, which are presented as follows:

$l1, l2$: Footing's dimensions.

$c1, c2$: Column's dimensions.

d: Depth of footing.

E_c: Modulus of elasticity of concrete.

E_s: Modulus of elasticity of soil.

The stress is the key point in this study, presented in Equation 1 (Hibbeler, 2011). Thus, the variables will be abbreviate as areas and dimension ratios. The variables are presented in Equation 2 and Equation 3

$\sigma = \frac{F}{A}$(1)

Where: F is force and A is area.

$A_c = c1 * c2$, where $c1$ and $c2$ are the columns dimensions (2)

A_c: Area of column section.

$A_f = l1 * l2$where$l1$ and $l2$ are the columns dimensions(3)

A_f: Area of footing section.'

Furthermore, the variables are assumed as the ratios: $\frac{A_c}{A_f}$ and $\frac{A_f}{d}$.

The modulus of elasticity is taken as a ratio of: modulus of elasticity of soil to the modulus of elasticity of concrete: $\frac{E_s}{E_c}$

The model is used to find the soil settlements (Δ_{soil}) and displacements of structure ($\Delta_{structure}$) and the total displacements (Δ_{total}). Then the results are normalized as the following ratios:

$$\frac{\Delta_{soil}}{\Delta_{total}}, \frac{\Delta_{structure}}{\Delta_{total}}$$

The stress is considered 6000KN/m^2, which represents the average service loads affecting a standard column in a resident structure. However, because the results are ratios of the total displacement in an elastic model, the value of stress has no significant effect on the ratios, which will be clarified in the following sections.

Because the soil dimensions used are specifically designed to dissipate the stress before reaching the lateral barriers, lateral end restrains are negligible. The bottom joints of the soil are restrained as pin support, to simulate a layer of rigid bedrock.

The shear friction forces at the interface between the footing and soil are very small and negligible (Touqan et all, 2017). Therefore, the interaction between soil and structure at the interface area is assumed continuous.

Analysis procedure

The variables are assumed to cover the diversity of dimensions used for column section and footing section. $\frac{A_c}{A_f}$ value ranges between 0.01 and 0.09, and $\frac{A_f}{d}$ value range between 6 and 20.

For a certain $\frac{A_c}{A_f}$ value, many $\frac{A_f}{d}$ values are tested. In addition, for each $\frac{A_c}{A_f}$ and $\frac{A_f}{d}$ value, models with different modulus of elasticity ratio ($\frac{E_s}{E_c}$) are tested.

For every tested model, values of Δ_{soil}, $\Delta_{structure}$ and Δ_{total} are obtained to calculate the ratios $\frac{\Delta_{soil}}{\Delta_{total}}$ and $\frac{\Delta_{structure}}{\Delta_{total}}$. The values of the calculated ratios are presented in a diagrams with $\frac{E_s}{E_c}$ as the abscissa, in order to test the effect of changing the variables $\frac{A_c}{A_f}$ and $\frac{A_f}{d}$.

Afterwards, equations can be concluded in term of the previously mentioned variables, which is used to eventually find the soil settlement.

As mentioned before, the used stress is 6000KN/m^2. The stress is assigned as pressure stress on the top face of the column.

Results and discussion:

Table (**1** is a sample of one test done for $\frac{A_c}{A_f} = 0.04$ and $\frac{A_f}{d} = 10$, where the results Δ_{soil}, $\Delta_{structure}$ and Δ_{total} are found, and the ratios $\frac{\Delta_{soil}}{\Delta_{total}}$ and $\frac{\Delta_{structure}}{\Delta_{total}}$ are calculated.

Table (1): Results from a test used in the study, presented as a sample.

$c1$	$c2$	A_c	$l1$	$l2$	A_f	$\frac{A_c}{A_f}$	d	$\frac{A_f}{d}$
0.6	0.4	0.24	3	2	6	0.04	0.6	10

$\frac{E_s}{E_c}$	Δ_{total}	Δ_{soil}	$\Delta_{structure}$	$\frac{\Delta_{structure}}{\Delta_{total}}$	$\frac{\Delta_{soil}}{\Delta_{total}}$
0.0002	76.2678	75.5183	0.7495	0.0098	0.9902
0.0020	8.5300	7.7800	0.7500	0.0879	0.9121
0.0201	1.6870	0.9380	0.7490	0.4440	0.5560
0.2012	0.8990	0.1480	0.7510	0.8354	0.1646
2.0116	0.7726	0.0215	0.7511	0.9722	0.0278
20.1162	0.7536	0.0025	0.7511	0.9967	0.0033

Upper and lower limits

Figure (**4** represents the curves using data from Table 1, which illustrates the limits of the rigid-soil flexible-structure phase, and the flexible-soil rigid-structure phase, and the conditions where these assumptions can be used.

Note that abscissa's scale is logarithmic in the figures, in order to illustrate the curves in more understandable form, to facility the study and the data fitting

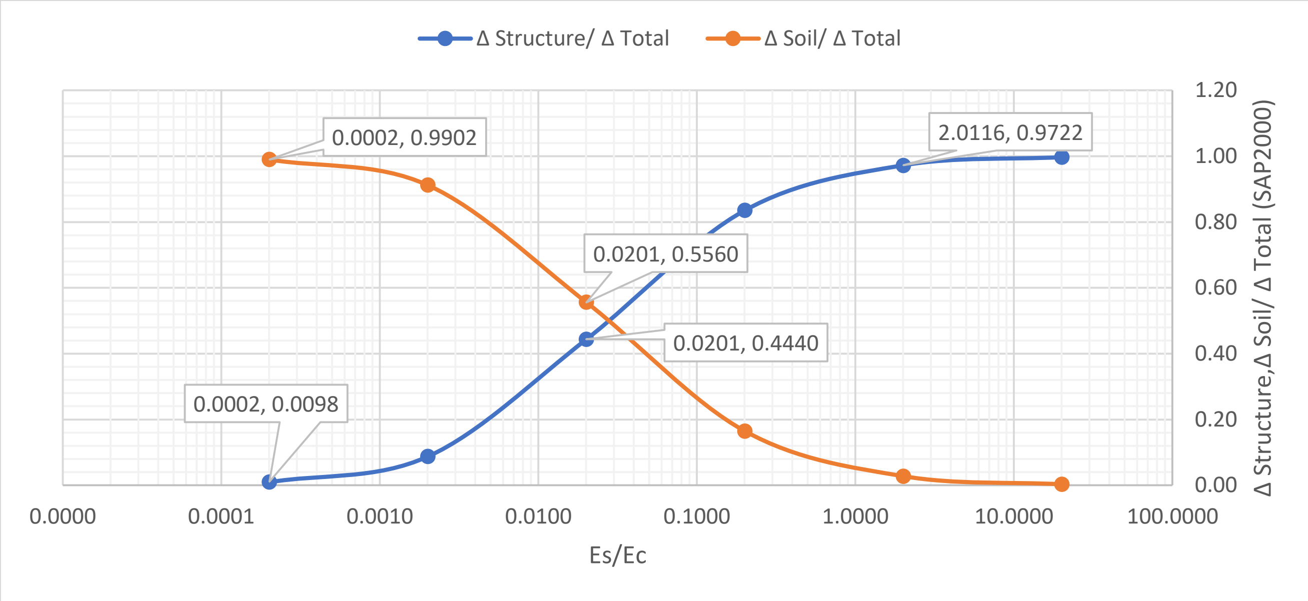

Figure (4): A sample of (Δ Soil/ Δ Total) and curves, for Ac/Af=0.04 and Af/d=10 versus Es/Ec.

As seen from Figure (**4**, $\frac{\Delta_{structure}}{\Delta_{total}}$ gives value of approximately 1 at a high modulus of elasticity ratio $\left(\frac{E_s}{E_c}\right)$ equals 2. Increasing the soil's modulus of elasticity will increase the rigidity of the soil. This means, at that point, the displacement of the structure is approximately 100% of the total displacement, so the rigid-soil flexible-structure assumption is valid.

On the contrast, when the soil has low modulus of elasticity value, like when the modulus of elasticity ratio $\frac{E_s}{E_c}$ equals approximately $2 * 10^{-3}$, $\frac{\Delta_{soil}}{\Delta_{total}}$ value is approximately 1. This means the flexible-soil rigid-structure can be used.

These limits are considered the upper and the lower boundaries. On the other hand, it can be noticed from the figure, the majority of the tested points are between these limits, more than $2 * 10^{-3}$ and less than 2. That means both the soil settlement and the displacement of structure are critical at this range.

The curves intersect each other between the modulus of elasticity range of 0.02 to 0.03. In this range the effect of soil structure interaction is so obvious, where $\frac{\Delta_{soil}}{\Delta_{total}}$ and $\frac{\Delta_{structure}}{\Delta_{total}}$ values are very close.

The effect of changing $\frac{A_c}{A_f}$ and $\frac{A_f}{d}$ on the curves

Figure (**5** shows the values of $\frac{A_c}{A_f}$ and $\frac{A_f}{d}$ used for the comparison, to find the effect of changing values of $\frac{A_c}{A_f}$ and $\frac{A_f}{d}$.

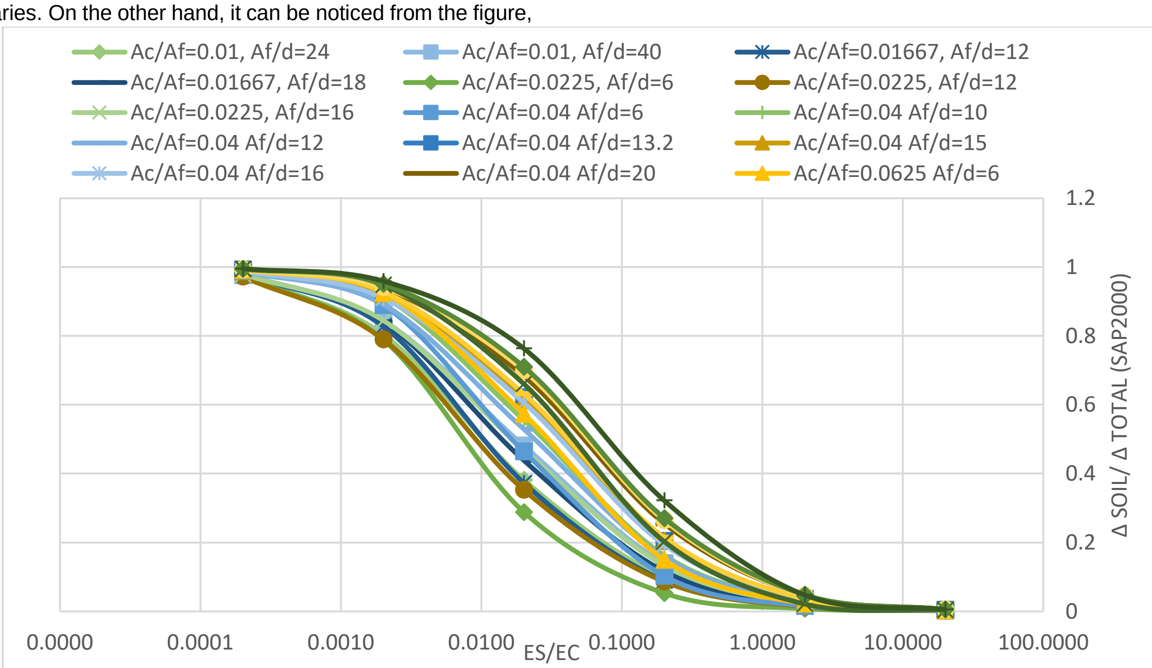

Figure (5): The change of (Δ Soil/ Δ Total) and (Δ Structure/ Δ Total) for different Af/d and Ac/Af values used in this study, versus Es/Ec.

Observations on $\frac{A_c}{A_f}$ effect on the curves

Figure (**6** to Figure (**8** shows the effect of changing $\frac{A_c}{A_f}$ value for a constant $\frac{A_f}{d}$ on the displacement of soil to total ratio $\frac{\Delta_{soil}}{\Delta_{total}}$, while Figure (**9** to Figure (**11** shows the effect of changing $\frac{A_c}{A_f}$ on $\frac{\Delta_{structure}}{\Delta_{total}}$, which is the mirror images of the Figure (**6** to Figure (**8**.

As can be noticed from the curves, increasing $\frac{A_c}{A_f}$ value increases $\frac{\Delta_{soil}}{\Delta_{total}}$ curve. For example, increasing $\frac{A_c}{A_f}$ value from 0.0225 to 0.04 increases $\frac{\Delta_{soil}}{\Delta_{total}}$ value at a $\frac{E_s}{E_c}$ value 0.02 from 28.8% to 46.5%, that is an increase of about 61.5%. Moreover, it is obvious that the increase occurred mainly between the $\frac{E_s}{E_c}$ values of 0.2 and 0.002, while the significance of the change decreases greatly for values more than 0.2 and less than 0.002. It is predictable because these values presents the upper and lower limits of the soil-structure interaction system.

Increasing $\frac{A_c}{A_f}$ value means increasing the area of the column or decreasing the area of the footing, which leads to the result of more rigid structure. Increasing the rigidity of the structure decreases its settlement, hence increases the share of the soil settlement from the total displacement.

The effect of changing $\frac{A_c}{A_f}$ value on $\frac{\Delta_{structure}}{\Delta_{total}}$ is the opposite. The increase of $\frac{A_c}{A_f}$ value decreases $\frac{\Delta_{structure}}{\Delta_{total}}$ for the same reasons previously explained.

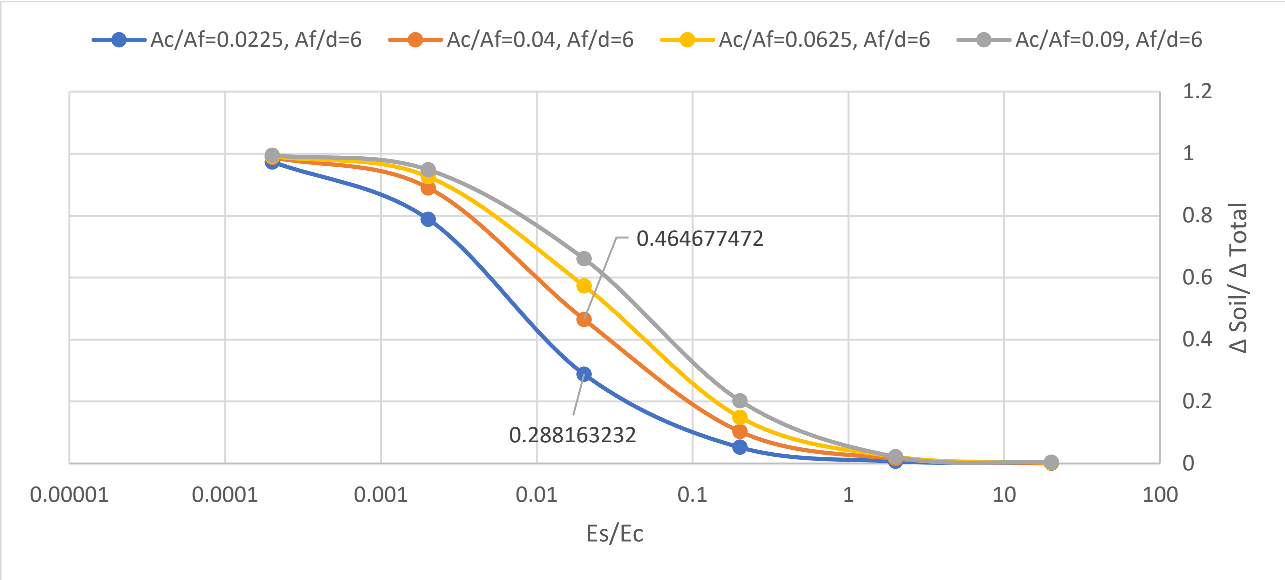

Figure (6): The change of (Δ Soil/ Δ Total) for Af/d=6 with different Ac/Af values, versus Es/Ec.

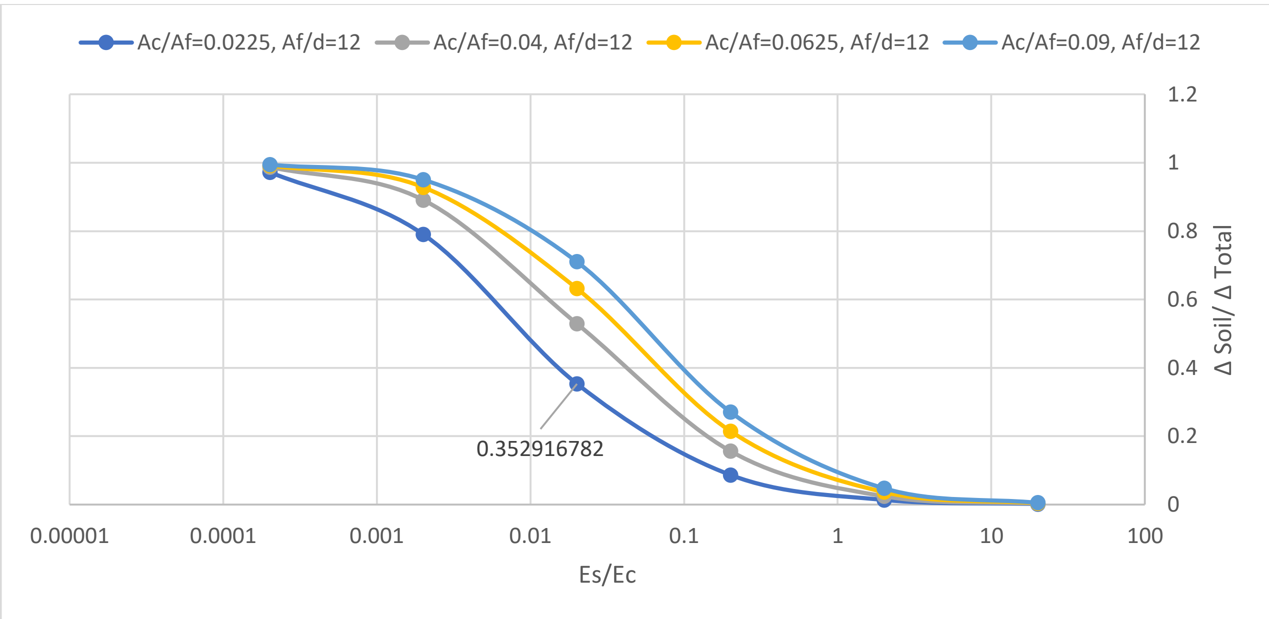

Figure (7): The change of (Δ Soil/ Δ Total) for Af/d=12 with different Ac/Af values, versus Es/Ec.

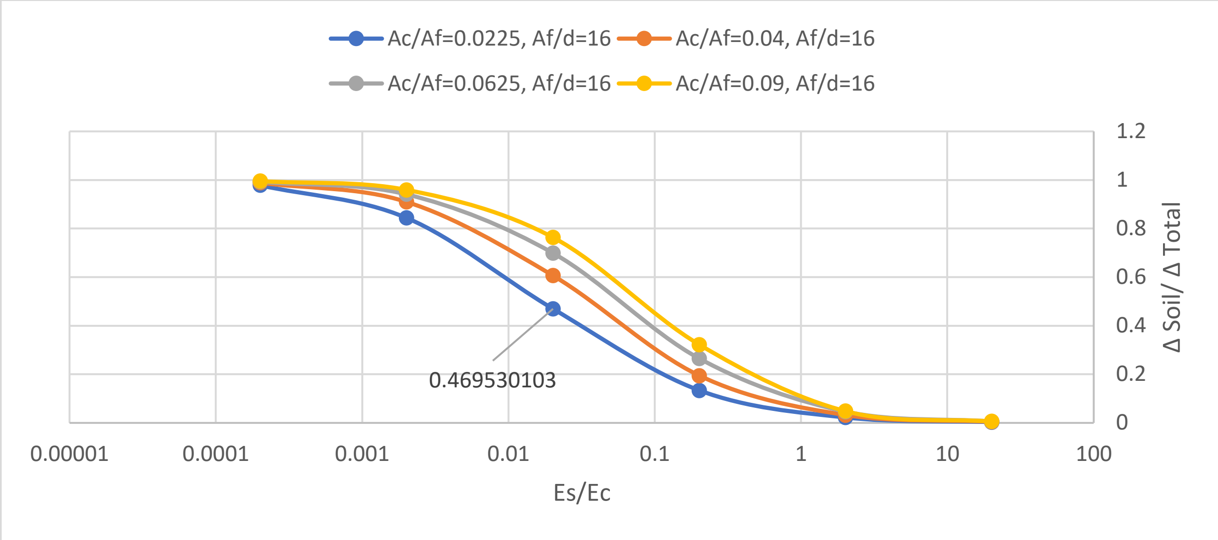

Figure (8): The change of (Δ Soil/ Δ Total) for Af/d=16 with different Ac/Af values, versus Es/Ec.

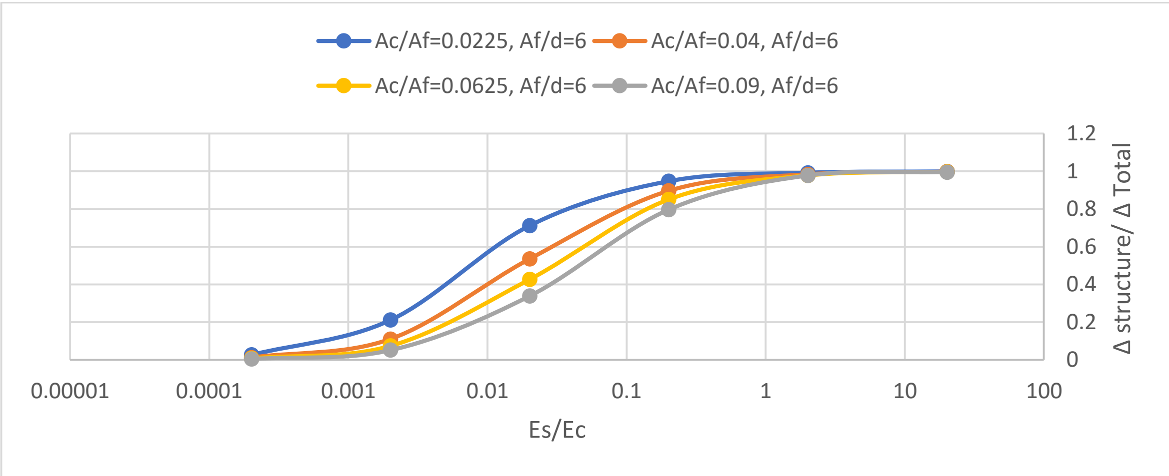

Figure (9): The change of (Δ Structure/ Δ Total) for Af/d=6 with different Ac/Af values, versus Es/Ec.

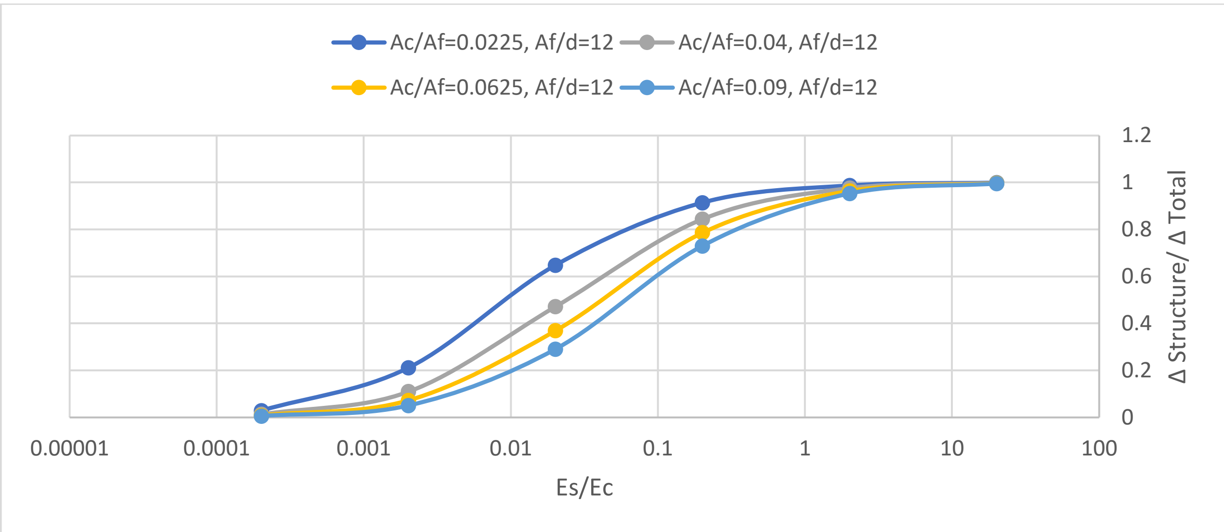

Figure (10): The change of (Δ Structure/ Δ Total) for Af/d=12 with different Ac/Af values, versus Es/Ec.

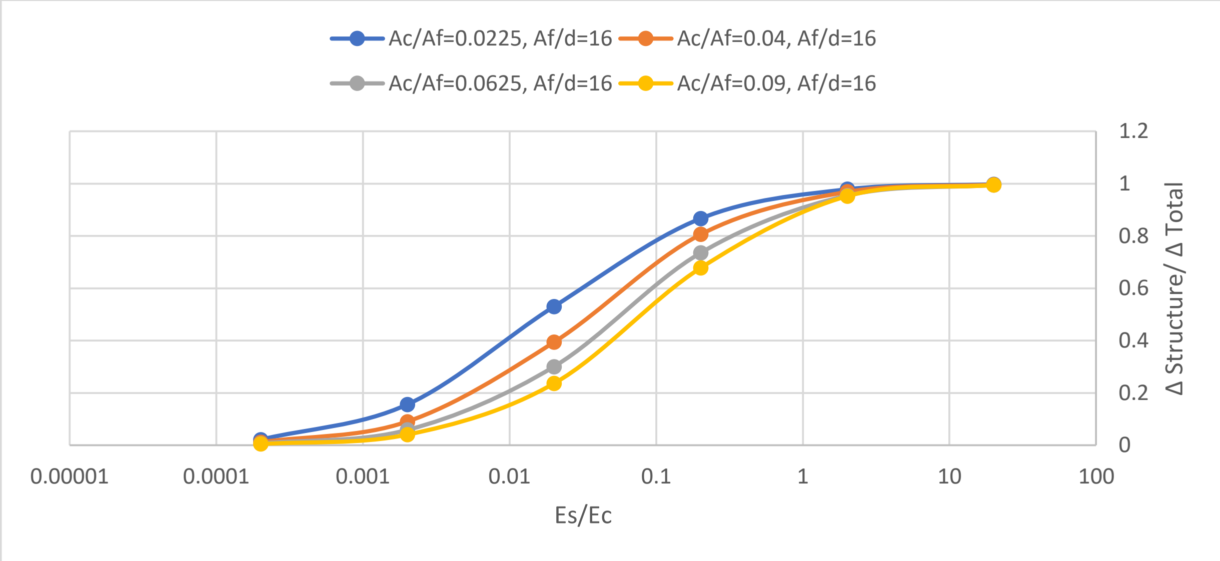

Figure (11): The change of (Δ Structure/ Δ Total) for Af/d=16 with different Ac/Af values, versus Es/Ec.

Observations on $\frac{A_f}{d}$ effect on the curves

From Figure (**6** to Figure (**11**, the effect of changing $\frac{A_f}{d}$ can be partially explained. The increase of $\frac{A_f}{d}$ value from 6 to 12 to 16 increases $\frac{\Delta_{soil}}{\Delta_{total}}$ for a certain $\frac{E_s}{E_c}$ value from 28.8% to 35.3% to 47% respectively, as evident in Figure 6 to Figure 8.

For further explanation, Figure (**12** shows a sample of tests with constant $\frac{A_c}{A_f}$ value of 0.04 and different $\frac{A_f}{d}$ values. The values of $\frac{\Delta_{soil}}{\Delta_{total}}$ from the figure assures the previous conclusion, which states that increasing $\frac{A_f}{d}$ value leads to the increasing of the value of $\frac{\Delta_{soil}}{\Delta_{total}}$.

Increasing $\frac{A_f}{d}$ can be done by increasing the footing's area or decreasing the depth. This change will decrease the stiffness of the footing, this will lead to a slight change of stress distribution from the footing surface affecting the soil. Moreover, when finding $\Delta_{structure}$ from the models, the displacement of the footing is included in the displacement of structure. This is obvious in Figure 12, because for a constant $\frac{A_c}{A_f}$ value the only way to change $\frac{A_f}{d}$ is by changing the depth of the footing. This change does not oppose the assumptions, because the depth of the footing is already included in $\frac{A_f}{d}$, thus the effect of the changing is already calculated.

The maximum different of $\frac{\Delta_{soil}}{\Delta_{total}}$ values between $\frac{A_f}{d}$ equals 6 and 19.8 is from 46.5% to 68.2%, which is 46.7%, still less that the difference between the two adjacent values of $\frac{A_c}{A_f}$ in Figure (**6** for example. This leads to the conclusion that $\frac{A_c}{A_f}$ ratio is much more significant than $\frac{A_f}{d}$.

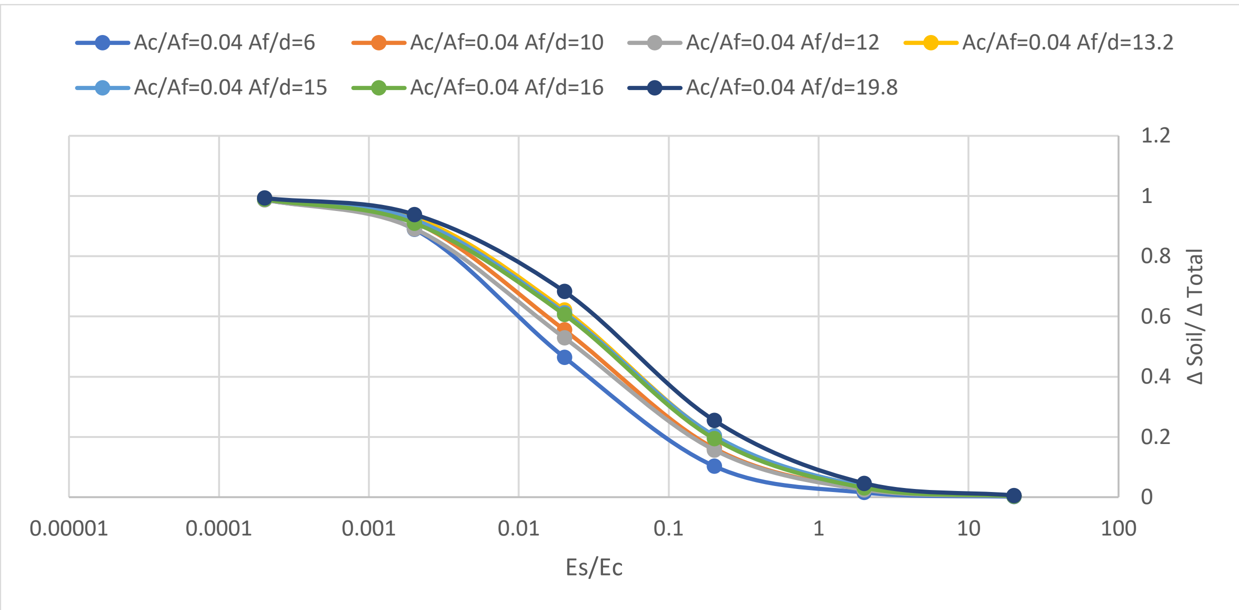

Figure (12): The change of (Δ Soil/ Δ Total) for Ac/Af equals 0.04, and different Af/d values, versus Es/Ec.

The slope of the curves

It is obvious that the slope of the tested model presented in Figure (**5** is approximately constant. The slope is the ratio the displacement ratio to modulus of elasticity ratio.

$$Slope = \frac{\Delta_{soil} * E_c}{\Delta_{total} * E_s}$$

While the stress is governed by Equation 4 (Hibbeler, 2011) the slope can be considered as an indication of the stress (Touqan et all, 2017). This leads to the assumption that the stress value is insignificant in this study. This behavior is expected because the soil structure interaction system is assumed fully elastic.

$\sigma = \varepsilon * E$(4)

Where: ε: The displacement.

E: The modulus of elasticity.

Comparing the effect of changing $\frac{A_c}{A_f}$ and $\frac{A_f}{d}$ values to the common design practice of footing.

When the structural engineer faces a major stress that exceeds the bearing capacity of soil, the natural choice the engineer will make is to increase the surface area of the footing, which is A_f, to distribute stress on larger area, which decreases soil settlement. Increasing A_f value means less $\frac{A_c}{A_f}$ value and more $\frac{A_f}{d}$ value which will affect $\frac{\Delta_{soil}}{\Delta_{total}}$. Decreasing $\frac{A_c}{A_f}$ will decrease $\frac{\Delta_{soil}}{\Delta_{total}}$ and increasing $\frac{A_f}{d}$ will increase $\frac{\Delta_{soil}}{\Delta_{total}}$. However, increasing the footing's area will eventually increase the depth of the footing to resist the shear and the punching shear, especially when the increase of the area affect the rigidity of the footing.

The increase of the depth will decrease $\frac{A_f}{d}$ value, decreasing $\frac{\Delta_{soil}}{\Delta_{total}}$.

Comparison between FEM's results and analytical method's results

In order to check the reliability of the results, the soil settlements are calculated and compared with the results obtained from SAP2000.

For the case this paper study, where the linear elastic model can be used, settlements under loaded surface, rigid or flexible can be found using Equation 6. (Holtz, 1991)

$S_i = C_s * q * B * \left(\frac{1-v^2}{E}\right)$(5)

Where: S_i: Immediate soil settlement.

C_s: shape and rigidity factor.

q: uniform stress on the footing.

B: characteristic dimension of the footing.

v: Poisson's ratio.

E: Soil elastic modulus.

Figure 13 shows a comparison between the results from SAP2000 and the results calculated from Equation 5, which from it is noted that the slope value R equals 0.998, which is an indication that the results are approximate.

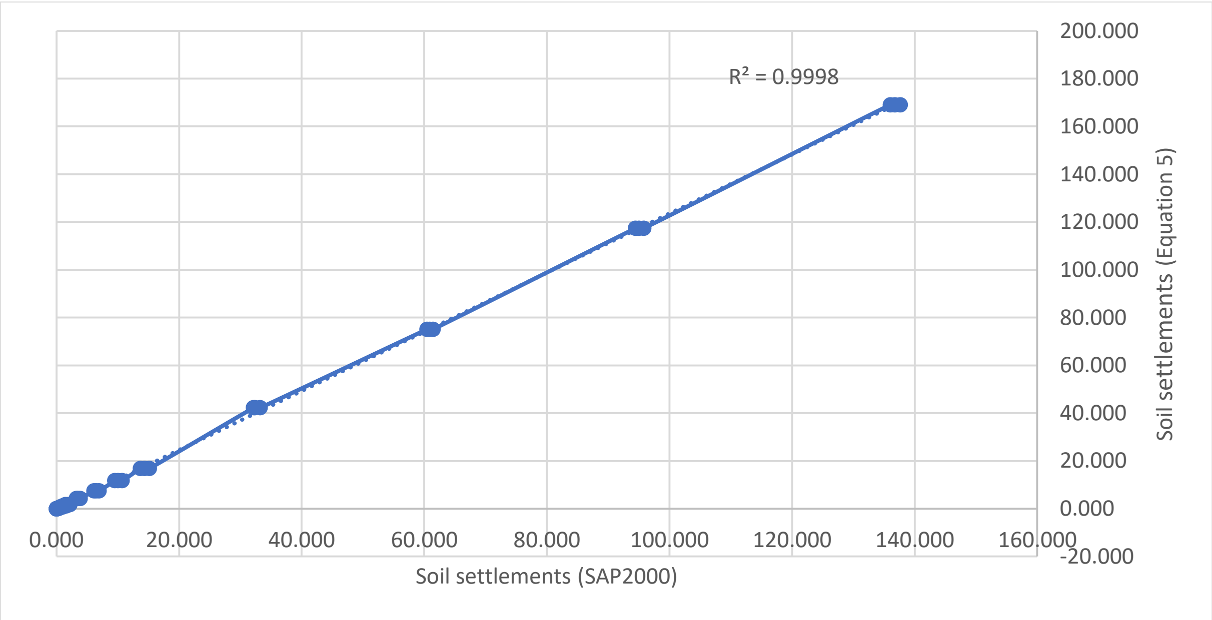

Figure (13): Comparison between soil settlements results from SAP2000 and Equation 5

Table 2 shows an example of comparison conducted between the settlement results from Holts (1991) both flexible and rigid, and the settlement from SAP2000 for $d = l_1/3$. It is obvious that for Es/Ec value of 0.020116 and 2.011621 the results from SAP2000 are approximately equal to the results from Holts (1991) flexible. Moreover, the results for Es/Ec less than 2.011621 to more than 0.002012, settlement values from SAP2000 is less than the results from Holts (1991) flexible but higher than the results from rigid. This means that both soil and structure is considered flexible objects. For Es/Ec less than 0.002012, the settlement is less than the rigid results, which means that the structure is very rigid, more than holts (1991) concluded. This matches the conclusion from the sections before.

Table (2): Comparison between the settlement from Holts (1991) flexible, rigid and the settlement from SAP2000 for $d = l_1/3$.

Es/Ec	S holts Flexible	S holts Rigid	S SAP2000 $d = l_1/3$
0.000201	150.15	113.295	94.00641
0.002012	15.015	11.3295	9.47211
0.020116	1.5015	1.13295	1.01766
0.201162	0.15015	0.113295	0.13282
2.011621	0.015015	0.01133	0.016401
20.11621	0.001502	0.001133	0.001418

However, when C_s is calculated from SAP2000 results, it was found that it changes with the change of soil rigidity. Figure 14 shows a comparison between the average C_s results calculated from SAP2000 and the values of C_s from (Holtz, 1991).

Three models were used in the comparison, where the depth of the footing d is taken as a ratio from the shortest footing dimension (l_1), where d equals $l_1/3$, $l_1/6$ and $l_1/8$.

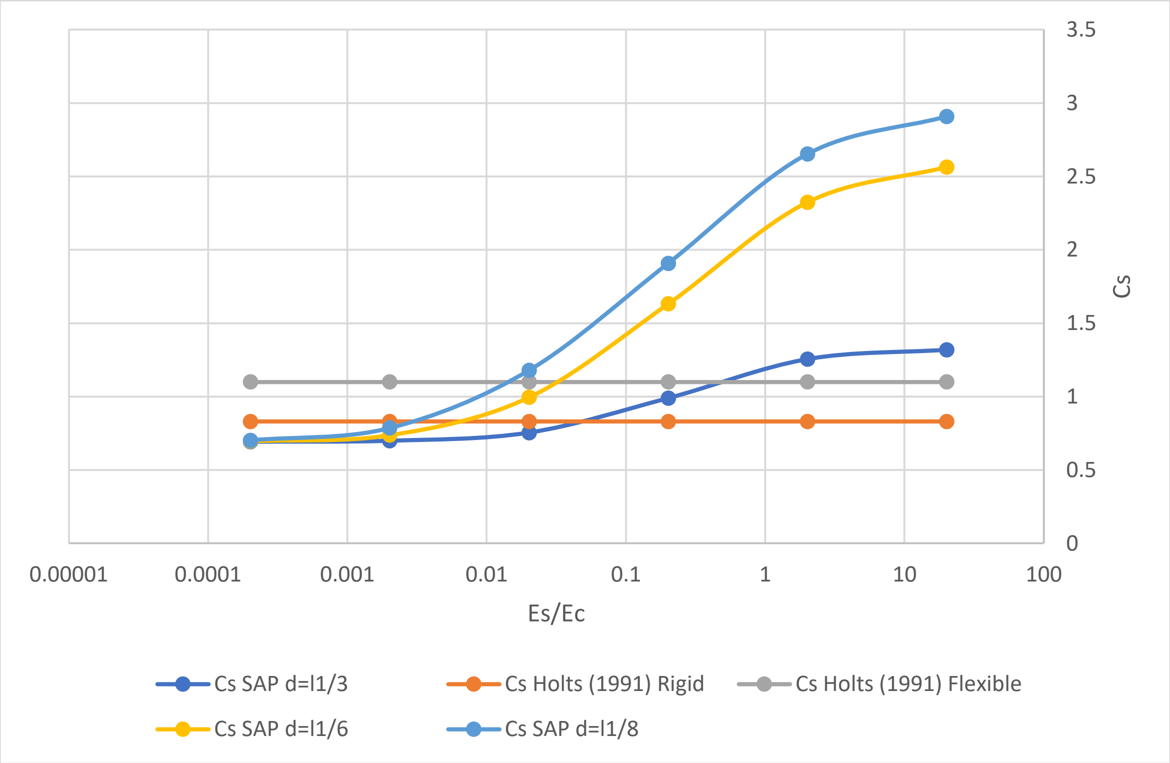

Figure (14): Comparison of Cs results from SAP2000 and Holtz (1991).

Figure 14 shows that increasing the soil rigidity will change the structural behavior to be more flexible, while reducing the soil rigidity will make the structure more rigid. Which means that the relative rigidity govern the soil structure interaction and not only the certain rigidity of the structure.

When d equals $l_1/3$, Cs values stars from 0.7 when the soil is weak, which indicates that the flexible soil – rigid structure is applicable. On the other hand, when the soil is very strong, Cs value from SAP2000 is 1.32, which indicates that the rigid soil – flexible structure is applicable.

Because the footing depth is reduced from $l_1/3$ to $l_1/8$ through $l_1/6$, Cs is increased significantly in the strong soil assumption with each reduce of the depth. This is expected because decreasing the footing depth will decrease the rigidity of the footing, which makes the structure very flexible. Decreasing the rigidity will make the structure start the flexible phase earlier on the Es/Ec scale, which will increase the Cs value significantly when the soil is very rigid. However, this reduce of rigidity does not change the value of Cs when the soil is very weak, because the assumed soil is very weak that even decreasing the rigidity of the footing does not change the value of Cs.

Data fitting

The aim of this study is to have a simple conceptual equation to find the soil settlement when the displacement of structure is known.

From Figure (**4** through Figure (**12**, it is clear that the curves behave like the S shape logistic curves.

(Verhulst, 1838) first derived the logistic curve general equation, which later has the simple form of Equation 5.

$$f(x) = \frac{x_1}{1+e^{-k(x-x_0)}} \quad (6)$$

Where:

x_1: The maximum value of the curve.

x_0: The x value of Sigmoid's midpoint.

k: The slope of the curve, which represents the logistic growth rate.

From the data collected and presented in the previous curves, Equation 6 and Equation 7 are concluded to find the values of $\frac{\Delta_{soil}}{\Delta_{total}}$ and $\frac{\Delta_{structure}}{\Delta_{total}}$ respectively.

$\frac{\Delta_{soil}}{\Delta_{total}} = \frac{1}{1+e^{2*(\log S_r - x_o)}}$(7)

$\frac{\Delta_{structure}}{\Delta_{total}} = 1 - \frac{1}{1+e^{2*(\log S_r - x_o)}}$(8)

Where:

$S_r = \frac{E_s}{E_c}$(9)

$x_o = \ln\left(\frac{A_c}{A_f}\right) * \alpha$(10)

$\alpha = 0.57 - 0.006 * \frac{A_f}{d}$ (11)

Reliability of the equations

To test the reliability of the equation, the ability of the equation to explain the upper and lower limits, hence the previously mentioned two main assumptions. Then, results from the equation will be presented versus the results from SAP2000, and by calculating the slope the accuracy of the results can be concluded. Finally, Cronbach's alpha method is used using sample of the results, to find the resulted alpha value and compare it with the acceptable value.

Upper and lower limits

For rigid-soil flexible-structure assumption, the soil is assumed rigid, which means that $\frac{E_s}{E_c}$ is approaching ∞, which means $\frac{\Delta_{structure}}{\Delta_{total}}$ value must be 1.Thus, Equation 7 is:

$$\frac{\Delta_{soil}}{\Delta_{total}} = \frac{1}{1+e^{2*(\log\infty - x_o)}} = 0$$

Therefore Equation 7 is:

$$\frac{\Delta_{structure}}{\Delta_{total}} = 1 - 0 = 1$$

On the other hand, flexible soil- rigid structure $\frac{E_s}{E_c}$ equals 0. Hence, the result of $\frac{\Delta_{soil}}{\Delta_{total}}$ from Equation 7 must equals 1.

$$\frac{\Delta_{soil}}{\Delta_{total}} = \frac{1}{1+e^{2*(\log 0 - x_o)}} = 1$$

In addition, $\frac{\Delta_{structure}}{\Delta_{total}}$ from Equation 8 will equal zero.

$$\frac{\Delta_{structure}}{\Delta_{total}} = 1 - 1 = 0$$

The results from Equations versus SAP2000

Figure (155 shows a comparison between the values of $\frac{\Delta_{soil}}{\Delta_{total}}$ from SAP2000 and the values resulted from Equation 7 for all the tested values of $\frac{A_c}{A_f}$ and $\frac{A_f}{d}$. In addition, Figure (**16** shows the comparison of $\frac{\Delta_{structure}}{\Delta_{total}}$ between SAP2000 and Equation 7 for all the tested values of $\frac{A_c}{A_f}$ and $\frac{A_f}{d}$. The slope can be an indication of the consistency of the results if the slope value is very close to 1. The slope value R from Figure 13 and Figure 14 is 0.998, which means the results from SAP2000 and Equation 7 and Equation 8 are almost identical.

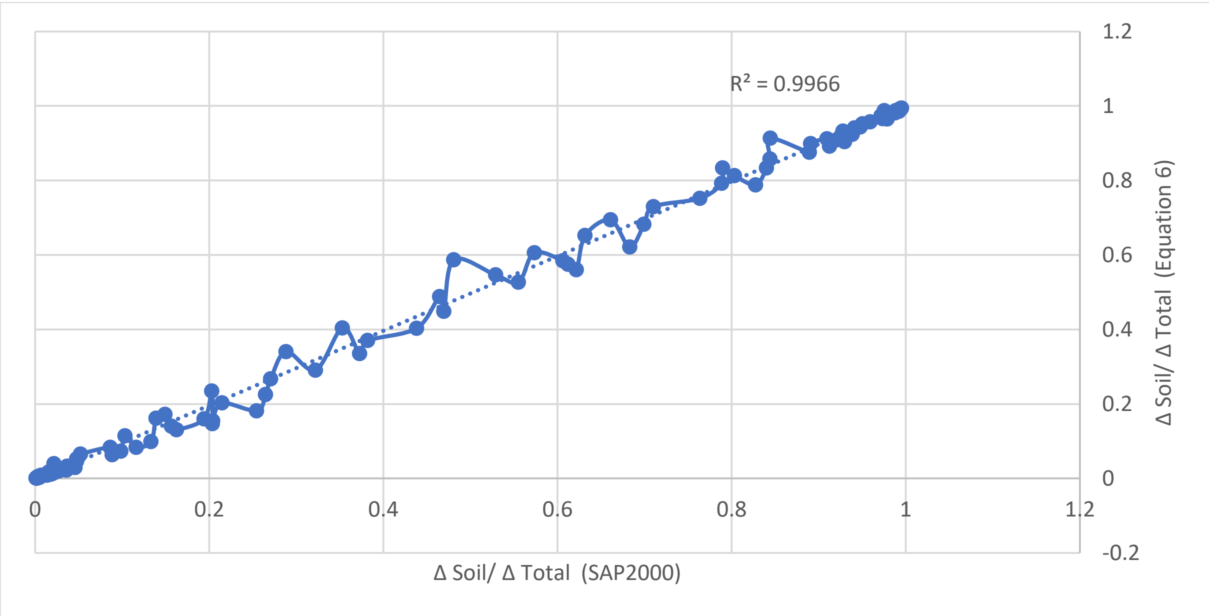

Figure (15): Comparing (Δ Soil/ Δ Total) value from SAP2000 versus the value from Equation 7.

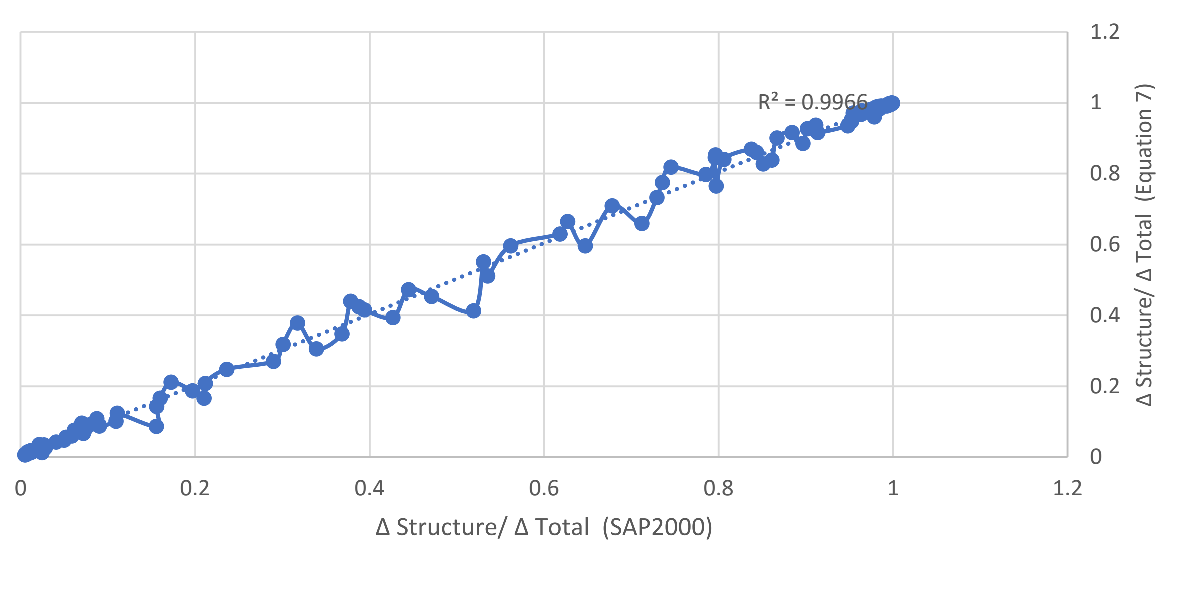

Figure (16): Comparing (Δ Structure/ Δ Total) value from SAP2000 versus the value from Equation 8.

Cronbach's Alpha

To assure that the results are reliable, the results from SAP2000 and Equation 7 and 8 are tested using Cronbach's alpha method. Cronbach's alpha is method used to test the reliability of results, which "calculated the average of the split half reliability coefficient obtained from all possible split halves" (Cho,2016).

(Cronbach, 1951) wrote a general equation to test the reliability of data, which is presented in Equation 12.

$$\alpha = \frac{n}{n-1} * \left(1 - \frac{\Sigma_i V_i}{V_t}\right); i = 1,2,3 \dots n \quad (12)$$

Where: V_t: The variance of test scores, V_i: The variance of item scores after weighting.

Using (A.C.I, 2018) equations, it was found that the majority of the designed foundations have values between 0.01 to 0.04 for $\frac{A_c}{A_f}$, and between 5 to 20 for $\frac{A_f}{d}$. Table 2 presents the data of the testes done in this study within these ranges for $\frac{\Delta_{structure}}{\Delta_{total}}$. It can be used as a comparison between the results from SAP2000 and Equation 7. After applying Cronbach's alpha on the data from Table 3, Equation 12 gives alpha value equals to 0.92, which is greater than 0.7, thus the results from Equation 6 and Equation 7 can be considered reliable (Cortina, 1993).

Table (3): Data from the majority testes done in this study for comparing (ΔStructure/ ΔTotal) values between the resulfrom SAP2000 and from Equation 8.

#	$\frac{\Delta_{structure}}{\Delta_{total}}$ (SAP2000)	$\frac{\Delta_{structure}}{\Delta_{total}}$ (Equation7)	#	$\frac{\Delta_{structure}}{\Delta_{total}}$ (SAP2000)	$\frac{\Delta_{structure}}{\Delta_{total}}$ (Equation7)	#	$\frac{\Delta_{structure}}{\Delta_{total}}$ (SAP2000)	$\frac{\Delta_{structure}}{\Delta_{total}}$ (Equation7)
1	0.0204	0.0263	26	0.1557	0.1423	51	0.3782	0.4395
2	0.1598	0.1666	27	0.5305	0.5508	52	0.7962	0.8528
3	0.5616	0.5963	28	0.8669	0.9006	53	0.9642	0.9772
4	0.8841	0.9161	29	0.9787	0.9853	54	0.9957	0.9968
5	0.9810	0.9878	30	0.9978	0.9980	55	0.0123	0.0188
6	0.9977	0.9983	31	0.0093	0.0134	56	0.1105	0.1240
7	0.0211	0.0351	32	0.0789	0.0909	57	0.5353	0.5112
8	0.1724	0.2117	33	0.3877	0.4249	58	0.8967	0.8854
9	0.6271	0.6649	34	0.7960	0.8452	59	0.9844	0.9828
10	0.9114	0.9361	35	0.9645	0.9758	60	0.9983	0.9976
11	0.9864	0.9909	36	0.9958	0.9967	61	0.0129	0.0150
12	0.9983	0.9988	37	0.0098	0.0161	62	0.1092	0.1010
13	0.0265	0.0342	38	0.0873	0.1082	63	0.4710	0.4536
14	0.2114	0.2074	39	0.4447	0.4726	64	0.8434	0.8598
15	0.7118	0.6591	40	0.8376	0.8688	65	0.9752	0.9784

#	$\frac{\Delta_{structure}}{\Delta_{total}}$ (SAP2000)	$\frac{\Delta_{structure}}{\Delta_{total}}$ (Equation7)	#	$\frac{\Delta_{structure}}{\Delta_{total}}$ (SAP2000)	$\frac{\Delta_{structure}}{\Delta_{total}}$ (Equation7)	#	$\frac{\Delta_{structure}}{\Delta_{total}}$ (SAP2000)	$\frac{\Delta_{structure}}{\Delta_{total}}$ (Equation7)
16	0.9478	0.9346	41	0.9730	0.9800	66	0.9971	0.9970
17	0.9927	0.9906	42	0.9958	0.9972	67	0.0113	0.0129
18	0.9992	0.9987	43	0.0071	0.0110	68	0.0902	0.0878
19	0.0280	0.0263	44	0.0614	0.0761	69	0.3938	0.4155
20	0.2101	0.1662	45	0.3171	0.3782	70	0.8060	0.8400
21	0.6471	0.5956	46	0.7455	0.8180	71	0.9687	0.9749
22	0.9136	0.9158	47	0.9541	0.9708	72	0.9965	0.9965
23	0.9865	0.9877	48	0.9946	0.9959			
24	0.9985	0.9983	49	0.0077	0.0142			
25	0.0212	0.0220	50	0.0700	0.0959			

Conclusion

For the same geometric, material, finite elements and model assumptions, the following points are concluded:

- The variable $\frac{A_c}{A_f}$ is very crucial for describing $\frac{\Delta_{soil}}{\Delta_{total}}$ and $\frac{\Delta_{structure}}{\Delta_{total}}$ behavior. It was found that increasing the value of $\frac{A_c}{A_f}$ will increase $\frac{\Delta_{soil}}{\Delta_{total}}$, therefore the soil settlements will increase. Increasing the area of the column or decreasing the area of the footing will increase the rigidity of the structure, which will decrease the displacement.

- Changing the variable $\frac{A_f}{d}$ changes the displacement ratios, but with less significance when compared with $\frac{A_c}{A_f}$. Increasing $\frac{A_f}{d}$ will increase the soil settlement, partially because changing this variable will change the total height of the structure, hence changing the whole displacement combination. In addition to the fact that changing this variable will affect the rigidity of the footing, resulting in a change in the stress distribution from the footing on the soil. Increasing $\frac{A_f}{d}$ means decreasing the depth of the footing, thus decreasing the rigidity of the structure.

- The limits for the both main two assumptions in soil-structure interaction were found in term of the modulus of elasticity ratio $\frac{E_s}{E_c}$. For ratios of values more than 2, the rigid-soil flexible-structure assumption is used. On the other hand, for ratios of values less than $2 * 10^{-3}$, the flexible-soil rigid-structure assumption is used. For $\frac{E_s}{E_c}$ more than $2 * 10^{-3}$ and less than 2, both soil and structure settlement is important, and both are considered flexible.

- The equations were deducted as follows:

$$\frac{\Delta_{soil}}{\Delta_{total}} = \frac{1}{1+e^{2*(\log S_r - x_o)}}$$

$$\frac{\Delta_{structure}}{\Delta_{total}} = 1 - \frac{1}{1+e^{2*(\log S_r - x_o)}}$$

Where: $S_r = \frac{E_s}{E_c}$, $x_o = \ln\left(\frac{A_c}{A_f}\right) * \alpha$ and $\alpha = 0.57 - 0.006 * \frac{A_f}{d}$

- The importance of the concluded equations comes from its simplicity. The structural engineer can use it during the design process to check if the soil settlement is within the acceptable rate, where the engineer can do adjustments on the footing's dimensions if the engineer found that the settlement is not within the range. Moreover, this method can be used for failure analysis related problems, to understand what was already occurred on the soil structure system.

Statement on ethics approval and consent

Not applicable.

Consent for publication

Not applicable.

Availability of data and materials

The raw data required to reproduce these findings are available in the body and illustrations of this manuscript.

Author's contribution

Theory, outline and final revision: Touqan.

Modeling, calculations, analysis, illustration and conclusion: Abu-Aladas.

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Conflicts of interest

The authors declare that there is no conflict of interest regarding the publication of this article

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