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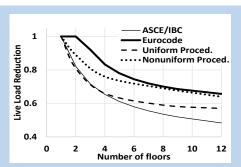
Reliability-Based Live Load Reduction Factors in Axial Force of **Columns in Residential and Office Buildings**

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Abstract: According to most building codes, the axial live load effect in columns of multi-story buildings may be reduced by some reduction factor. The justification for such a reduction is that there is a small probability for having all spans and floors of the building occupied with a nominal live load value. While this justification is jointly adopted by all buildings codes and standards, there are significant variations in the degree of reduction as allowed by these building codes. A probable cause for such significant variations is that these reduction factors were not based on reliability analyses, but rather on some extrapolation of different stochastic live load models to multi-story buildings. This study re-establishes the live load reduction factors based on reliability analysis, specifically, by maintaining the same reliability index for the columns. To achieve this goal, Monte-Carlo technique is utilized through the finite element software "ANSYS" on several thousand models of multi-story buildings. Live load, geometric and material variabilities are taken into consideration at many levels



of statistical representation. The resulting stochastic F.E. analysis gives the probabilistic distribution of axial live load in columns, which is then used to re-calculate the characteristic "nominal" value for the axial live load based on a reliability index. The obtained "nominal" value of axial live load in columns is then compared to the axial live load value obtained using nominal load value as given in codes and standards distributed uniformly over all spans and floors of the building. Based on this comparison, the reduction factor is readily defined as the ratio between the two values. The resulting reduction factors are then compared to those given in codes and standards and conclusions are then drawn.

Keywords: Stochastic Finite Element, Reliability Analysis, Live Load Reduction Factor, Design Codes.

Introduction

Most building codes allow reduction in live load at ultimate state design of columns in multi-story buildings. This is generally justified by the fact that there is a small probability for having all spans in the building simultaneously occupied by the codespecified nominal value of live load. The reduction generally depends on the tributary or influence area and the number of floors supported by the column in consideration.

Different building codes allow for different amount of reduction in live load. The derivation of such reduction is mainly based on statistical models and is not directly linked to the structural reliability or structural behavior. The stochastic process for generating the live load statistics is simply extended from spatial variation to multi-story case.

Corotis (Corotis) presented a statistical model that describes the live loading as a function of the degree of specification of building use during the design phase. That model provides the classification of mean, variance, and correlation in live load as a function of use, and leads to suggestions concerning future live load surveys. Idota (IDOTA) used the model by Corotis to generate statistics for typical floors with multiple columns in office buildings, and used the resulting information with actual data from surveys of normal live load (sustained load) in multi-story office buildings to establish live load reduction factors in axial loads of the columns based on their loaded areas and number of floors. The study was mainly statistical and the correlation

between each floor and between each non-adjacent columns was taken from Corotis model. Another study by McGuire and Cornell (McGuire and Cornell) focused on the effects of loaded area, shape of influence surface, and division into independently acting tenants on the live load variability. The authors of that study suggested a simple probabilistic model for live load in office buildings.

A recent study by Soomro et al. (Soomro, Khoso and Ali) showed through experiments and realistic data collection on residential multi-story buildings that a safety margin can still be maintained even when the live load is reduced to two-third of the code-specified values. The authors collected data on existing code-designed buildings which strictly followed requirements and then through experiments and calculations found the required live load based on their actual capacity. The results of the study showed that the live load given in design codes can be reduced to two-third of the code-value without losing safety margin.

A study by Ziemian and MacGuire (Ziemian and MacGuire) presented an approach for incorporating the live load reduction factor in the structural analysis of frames while maintaining joint equilibrium within the frame. The method however is not based on probabilistic or reliability analyses.

Another study (Piplodiy, a and A) reviewed the importance of considering live load reduction factors in the design of highimportance buildings under seismic loads.

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A recent review on the stochastic models of live loads in buildings is presented in Reference (Costa and Beck). The review showed that the reliability index computed based on many live load stochastic models falls below the prescribed 50-year target reliability index of the Eurocode. This study highlighted the need to re-calibrate code given live load values based on maintaining reliability indices.

Most of the previous studies used probabilistic models for the spatial and temporal variation live load to deduce the intensities of live loads on model floors with various areas, which were then used to compute the nominal percentile of live load on those floors.

To the knowledge of the authors of this study, none has done stochastic finite element simulations of framed structures to directly generate the statistics for the axial live load in columns. Such an approach is considered a novelty since it presents realistic simulation of the variability of the loading over the entire frame and gives the effect of loading on non-adjacent floors on columns more accurately due to the direct structural modelling of all spans and floors. The approach involves generating a random distribution of the axial force in a column in a building. Then the design value of the axial force (P^r_Ω) is computed from the random distribution based on a selected value of the reliability index (β) typically used for reliability studies. Further details on the approach are discussed in the Methodology of Analysis Section.

Live Load Reduction Factors in Codes and Standards

According to ASCE 07-10 standard (7-05), and as also adopted in the International Building Code (IBC 2021) (Council), the following formula is used to estimate the live load reduction factor:

$$(L/L_o)_{ASCE,IBC} = 0.25 + \frac{4.57}{\sqrt{K_{LL}A_T}}$$
 (1)

L = Reduced design live load per m² of area supported by the member.

 L_o = Unreduced design live load per m² of area supported by the member, not more than 4.79 kN/m²

 A_T = Tributary area in m²

 K_{LL} = Live load element factor ($K_{LL}*A_T$ is called the influence area, and it must be greater than 37.16 m²)

 L/L_{\circ} should not be less than 0.5 for members supporting one floor and not less than 0.4 for members supporting two or more floors

 K_{LL} ranges from 1 to 4 depending on location of the member and nature of loading transferred from the slabs.

In the Eurocode (Standardization), the live reduction factor is given as the multiplication of two independent reduction factors: the first factor is based on the loaded area carried by the member, and the other is based on the number of floors supported by the member. For the area-based reduction factor, the formula is given as:

$$(L/L_o)_{ECA} = \frac{5}{7}\psi_o + \frac{10}{A}$$
 (2)

where: A is the loaded area and ψ_0 is a combination factor according to EN 1990 Annex A1 Table A1.1, and equals 0.7 for most occupancy categories. For the floor-based reduction factor, the following formula is given:

$$(L/L_0)_{ECn} = \frac{2+(n-1)\psi_0}{n} \le 1.0$$
 (3)

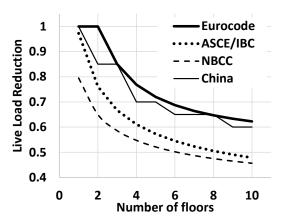
where n = number of stories (>2) above the loaded structural element. The live load reduction factor can then be computed as the multiplication of the two factors.

In the Canadian National Building Code (Codes), the formula for the reduction factor is:

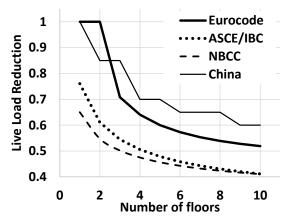
$$(L/L_o)_{NBCC} = 0.3 + \sqrt{\frac{9.8}{B}}$$
 (4)

where: B is the sum of all areas supported by the element, which must be greater than 20 m^2 .

In the Chinese building code (Construction), the reduction factors are given in a table depending on the number of floors only.



a) Loaded Area = 40 m²



b) Loaded Area = 80 m²

Figure (1): Comparison for the live load reduction factor given in different standards and building codes.

A comparison is shown in Figure 1 for the variation in the reduction factor as allowed in different building codes and standards. The reason for such variation can be attributed to variations in the statistics of live loads, particularly the extreme value of live loads, the statistical superposition rules, and for the differences in the driving and motivating philosophy behind the reduction itself.

The size of variation between building codes as shown in Figure 1 is significant and may have a relatively large effect on the design of structural systems. For example, the interior column in a ten-story building has a live load reduction factor of about 0.4 in the Canadian and American codes while it has a factor of 0.6 in the Chinse code (which is about 50% more than the value computed based on the ASCE). These large differences motivate a more reliable and structurally logical approach that would unify the philosophy and ground for deriving reduction factors for live loads.

Methodology of Analysis

In order to arrive at a reasonable and reliable value for the reduction factors, a logical and reproducible method must be adopted. The method is outlined below with the details of each step explained in the subsequent sections.

At first, a target reliability index is selected for the column or member under study. A representative model for multi-story buildings is selected. The building is loaded with live load through the random process that will be explained later. The results of the FE analysis give a probabilistic distribution of the axial live load (P_x) on the column. The concern now is to look for a design value (P_Ω^r) from this realistic distribution for live axial load that would maintain the same reliability index chosen for the column. Since the designer will not have the luxury of generating such random distribution of axial load, it is assumed that the nominal value for the live load as specified by the relevant building code, is imposed uniformly on the entire building, and this produces a nominal value for the axial load (P_n) . Based on this procedure, the live load reduction factor can be defined as:

$$(L/L_o)_{Proposed} = \frac{P_\Omega^r}{P_D}$$
 (5)

This procedure has the advantage of being logical and reasonable and represents the realism in the process of loading itself. These attributes make it reproducible and flexible to fit for various design categories and requirements.

The specifics related to the procedure proposed above will be explained in the following sections.

Target Reliability Index

The reliability index (β) for a column gives an indication of the acceptable probability of failure for that column. The failure in our analysis is defined by the instant when the maximum expected axial load from the random live load distribution exceeds the value obtained using uniform nominal load as specified by the building code. Typically, a minimum value of the reliability index for columns ranges between 3 and 4. In the Eurocode, a specific value of $\beta=3.8$ is used for reinforced concrete columns, while for the ACI 318 $\beta=3.5$ is considered satisfactory. In our analysis, a value of 3.5 is used for β . If the ratio between the random axial force and the force from nominal distribution (P_x/P_n) follows a normal distribution, then the reduction factor represents the 99.98 percentile of the standardized normal distribution. This value corresponds to a reliability index β of 3.5 as adopted by ACI 318.

Finite Element Model

A two-dimensional model is created for a framed office building. The columns and beams are designed using LRFD method as per the ACI standards. Since almost all design codes allow for elastic first-order analysis to be used for design of beams and columns within the frame, it is sufficient to use 3-noded elastic beam elements in the analysis, with 6 DOF per node. Each member is divided into 20 elements to produce sufficiently accurate results. Different types of frames: 2-bay, 3-bay and 4-bay frames are constructed with variable floors starting from 1 floor up to 20 floors for each frame. For each frame, spans of 4m, 6m, and 8m are assumed for the beams and a floor height of 3m is considered.

Statistical Data

Generally, the live load on residential and office buildings is thought to be comprised of two components: A transient "Arbitrary-point-in-time" component, also known as sustained live load component "Ls", and an extreme (extraordinary) component "Le", also known as intermittent load. The maximum possible live load is defined as the maximum load resulting from combining the two components together. Arbitrary-point-in-time live load affecting the construction at any time is determined as a result of field researches and direct surveys since it is recurring on daily basis in the structure. However, there can be moments when the live load attains extremely high values due to unexpected loading scenario, such as a renovation or sudden

change in the functionality of the rooms, or people crowding, etc. Field surveys generally cannot capture such extreme scenarios due to their short time frame and unexpectedness. Therefore, the extreme values are determined based on an assumed models or statistical distributions and then added to the sustained live load to generate the maximum possible value for live load.

The extraordinary live load and the maximum live load can be represented by Type 1 extreme value distribution while for the sustained component of live load a Gamma distribution would suffice (Ellingwood, Galambos and MacGregor) (Kumar) (Komurcu and Yucemen) (McGuire and Cornell) (Firat and Yucemen). The combined maximum loading seems to be sufficiently represented by a Gamma probability distribution. Even though most of the data is available for office buildings, their statistical characteristics seem to not differ much from data on multi-story residential and retail establishments [1].

Table 1 shows collected field data on the arbitrary-point-in-time (or sustained) live loads in residential buildings. The table is adapted from Ref (Firat and Yucemen). Overall, the mean value for the sustained live loads ranged from 0.31 kN/m² to 0.68 kN/m² and the standard deviation from 0.15 to 0.66. The data in the table indicates that both the mean value and the standard deviation for the sustained live load seem to get smaller for larger room areas.

Data on maximum live loads can be found in Ref. (Ellingwood, Galambos and MacGregor) which collected data from (McGuire and Cornell) (Ellingwood and Culver, Analysis of live loads in office buildings.) (Sentler) (Chalk and R. B. Corotis) for both residential and office buildings alike. Based on these data the ratio of the mean value to the nominal value for the maximum live loads ranged from 1.11 to 1.38, and the coefficient of variation ranged 0.09 to 0.18.

In our study, two procedures are used; namely: uniform approach and non-uniform approach. In the uniform approach, a uniform live load is applied on each span of the structure with the value of the load randomly selected from a maximum live load distribution, which was assumed to follow a Gamma distribution with a mean value of 1.35 kN/m² and a variance of (0.25)² kN/m² (COV = 0.185). These values are obtained by combining the sustained live load and the extraordinary live load distributions. It should be noted that for the computed parameters of the maximum live load distribution, the 98 percentile gives the nominal uniform live load of 1.95 kN/m2 which is very close to that used in residential buildings as given in the ASCE07. The sustained live load is also assumed to follow a Gamma distribution, with a mean of 0.5 kN/m2 and variance of (0.2)2 kN/m2. For the extraordinary live load, it is assumed to follow a Type 1 Extreme Value distribution, with a mean of 1.1 kN/m² and a variance of (0.05)2 kN/m2. These statistics come from the recommendations in Ref. (Thurmond, Woeste and W.) and quoted from several Canadian researchers therein.

Table (1): Field data on sustained live loads in residential buildings (Firat and Yucemen).

Study	Room Floor Area (m2)	Average Live Load kN/m2	Standard Deviation
Mitchell and Woodgate (1971)	2.4	0.66	0.65
	5.2	0.64	0.53
	14.0	0.62	0.43
	31.2	0.61	0.34
	58	0.59	0.30
	111.3	0.58	0.26
	192.4	0.56	0.21
Choi (1992)	0-5	0.5	0.66
	5-10	0.62	0.64
	10-20	0.55	0.47
	20-40	0.45	0.53
	40-80	0.43	0.45
	80-	0.51	0.41

Study	Room Floor Area (m2)	Average Live Load kN/m2	Standard Deviation
	0-8	0.68	0.41
	8-16	0.60	0.32
	16-24	0.50	0.36
	24-32	0.50	0.29
Kumar (2002)	32-40	0.47	0.26
Kumar (2002)	40-48	0.45	0.24
	48-56	0.45	0.25
	64-72	0.46	0.15
	72-80	0.46	0.19
	80-	0.31	0.20

Live Load Sequencing

There have been many attempts to combine information about sustained and extraordinary live loads (see Ref. (Thurmond, Woeste and W.) for a discussion on several methodologies, and also see (Chalk and R. B. Corotis) (Sentler) (Wen)). All these methods relied on a kind of super imposing the data from sustained and transient live loads to generate maximum live loads and then how to predict the statistics of the resulting distribution. The prime difference in their assumptions is the temporal progression for each distribution and type of correlation (if existed) between them all.

As discussed earlier, we proposed two procedures to populate the spans of the beams with live loads in any frame. The first, which will be henceforth called "uniform procedure" is to assume the live load to be uniformly distributed but distinct in each span, with a value drawn from the Gamma distribution representing maximum live load value in that span. The second procedure, henceforth called "non-uniform procedure", is to assume a distinct variation in live load values within each span. The span of each beam is divided into segments of 1 meter length. At first, random values from the sustained live load are distributed over the segments of each span. Then, the extraordinary live load values are generated for each segment and then super imposed on the sustained values to produce a maximum value for the live load. The two procedures are shown in Figure 2.

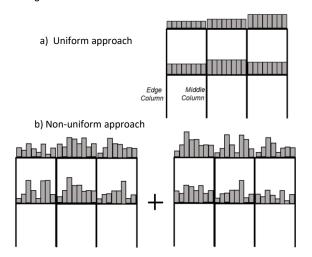


Figure (2): Approaches used for live load random sequencing representing a single simulation.

Stochastic Simulations and Results

In total, more than 5500 random simulations are conducted using finite element program ANSYS. The plane frames are parameterized in term of number of bays, span lengths, number of floors, and nature of loading distribution. Analysis included frames of 2, 3 and 4 bays with beam spans of 4m, 6m, and 8m, and number of floors of 1, 2, 4, 8, 12, and 16 floors. For each type of frame with specified number of bays, floors and span length, 50 probabilistic simulations are conducted where the live load is varied according to the methods of loading. Each simulated frame is analyzed three independent times: the first is

using the uniform approach; the second is using the non-uniform approach for live load sequencing, and the third is the benchmark case wherein the nominal live load value specified in the ASCE 07 building code is imposed uniformly over all the spans of the frame.

In each single simulation, the axial live loads in the edge and middle columns are recorded. Samples from the results of the simulations are shown in Figure 3. In the figure, the horizontal axis represents the ratio of the obtained axial forces from the random simulations (P_x) to the value obtained using nominal load value uniformly distributed over all spans (P_n) . This ratio in effect represents the actual reduction factor for the axial live load for each simulation. The live load reduction factor for the axial force can be established from these results by taking $P_x = P_{\Omega}^r$, where P_{Ω}^r is the value that produces the reliability index $\beta = 3.5$. The distribution of P_x/P_{Ω}^r is fitted to the best distribution of either Gamma, or Log-Normal or Normal, and then the inverse function for the distribution is used to find $P_{\Omega}^r = \Phi(z = 3.5)$

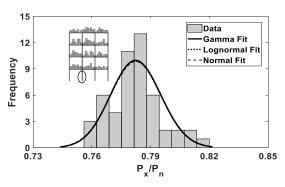
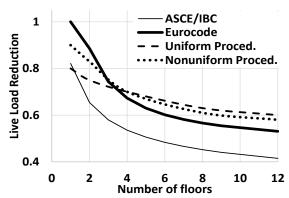
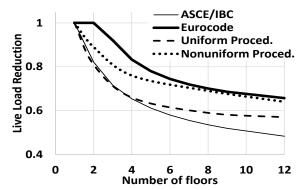


Figure (3): Variation in axial live load in middle column.

Using the results from each set of simulations, a reduction factor can be calculated for each combination of the parameters. These results are shown in Figure 4 below.



a) Middle column (Loaded area = 64m²)



b) Edge column (Loaded area = 32m2)

Figure (4): Axial live load reduction factor for 3-Bay frame of 8 m span for a) middle column and b) edge column.

Results and Discussion

The fitting of the data points was done using custom curve fitting tool available in MATLAB. The independent variable was chosen to be the tributary area of the column, which is defined to be the accumulative loaded area on the column from all floors above, thus incorporating the number of floors in the proposed fitted equation. The data points from all the simulations are shown in Figure 5. In total, 108 independent data points resulted from the overall stochastic simulations are shown in Figure 5.

The form of the equation was chosen as:

$$(L/L_o)_{Proposed} = a + \frac{b}{A_T + c} \le 1.0$$
 (6)

where $(L/L_o)_{Proposed}$ is the proposed reduction of the axial live load in column, A_T = tributary area on the column and equals the loaded area times the number of floors above the column. The coefficients a, b, and c are fitting coefficients whose raw values are a = 0.4898 \in (0.4615, 0.5182), b = 44.17 \in (33.9, 54.44), and c = 68.96 \in (51.68, 86.24). The given intervals represent 95% confidence bounds for each coefficient. For practical reasons, the values are rounded into simpler numerals, specifically: a = 0.5, b = 44 m², and c = 70 m², and the resulting equation is plotted using the rounded coefficients in Figure 5. The equation produces a total of root mean square error of 4.3%. Figure 6 shows the sensitivity of the proposed live load reduction equation to changes in the coefficients a, b and c. It can be seen that a small increase in the proposed coefficient produces a small change in the proposed equation results.

To compare the results from the existing equation with those given in current codes, the live load reduction factor is computed using the proposed equation, ASCE code, Eurocode the Canadian code and the Chinese code for an interior column in a building that has 8 stories and carries a tributary area of 30 m2 in each story. The results for the live load reduction factor (LLRF) are given in Table 2.

Table (2): Comparison between LLRF from the Proposed Equation and Current Codes.

Method	ASCE	Eurocode	NBCC	China	Proposed Approach
LLRF	0.397	0.821	0.502	0.65	0.642

It can be seen that there are large differences among the computed values. These large differences may have large impact on the design of the column.

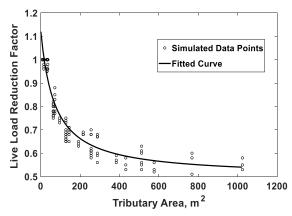


Figure (5): Curve fitting of all resulting data points.

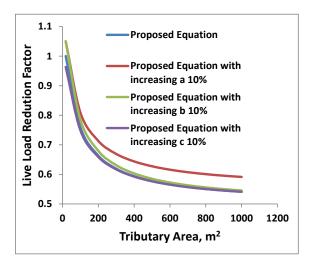


Figure (6): Sensitivity of the Proposed Equation to Changes in a, b and c

Conclusion

In this study, the live load reduction factor for the axial force in the columns is re-established based on reliability analysis. Stochastic finite element simulations were conducted and the variability in live load is directly included in the analysis. The results showed high variability in the reduction factor and also high variability among design codes and standards. Based on the stochastic results, a model that predicts the reduction factor is obtained through data fitting of the results. The proposed model is statistically-based and provides a more reliable approach with a unified philosophy for the live load reduction factor

Disclosure Statement

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- Author's contribution: The two authors confirm contribution to the paper as follows: idea conception and design: Monther. Data analysis, modeling and validation: Mahmoud. Theoretical derivation: Monther. Draft manuscript preparation: Mahmoud. The two authors reviewed the results and approved the final version of the manuscript.
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