An-Najah University Journal for Research – A
Natural Sciences

Application of Adiabatic Process for Decreasing Energy Loss in Discharging A Capacitor

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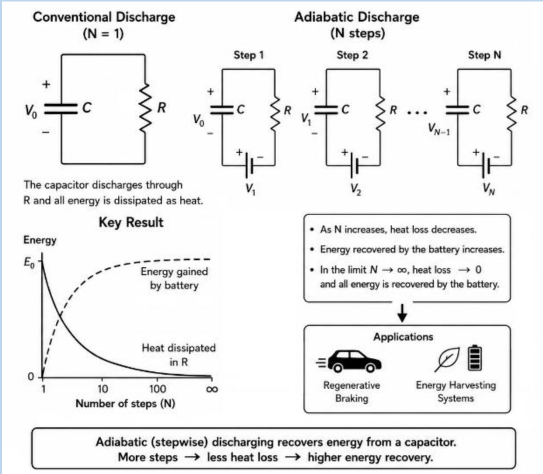
Type: Full Article. Received: 26th Mar. 2026, Accepted: 10th May. 2026

Received Accepted, In Press

Abstract: This paper presents an application of adiabatic technique to the process of discharging a capacitor in an *RC* circuit. In such a process, a fully charged capacitor is allowed to release its stored charge through an arbitrary number of steps. Each step is controlled by including a battery whose voltage is less than, and opposite in polarity to the voltage on the capacitor. Theoretical derivation of the heat energy dissipated in the resistor and the energy gained by the battery is presented and analyzed. It is shown that the dissipated heat energy decreases as the step number increases but constant across all steps. Our results also show that the initial energy stored in the capacitor is distributed as heat energy and energy gained by the battery. Moreover, as the step number increases, the energy gain by the battery increases and the heat energy decreases and in the limit as the number of steps approaches infinity the heat energy vanishes and thus all the initial energy in the capacitor has been gained by the battery. In addition, our theoretical adiabatic method is tested experimentally and consistent results are obtained. This method of adiabatic discharging for large N recovers up to 99% of the stored energy, offering applications in regenerative braking and energy harvesting systems.



Keywords: Adiabatic energy recovery; Sustainable electronics; Energy-efficient RC circuits; Low-power circuit design; Energy harvesting; Regenerative braking; Green electronics.

Introduction

Energy flow in RC circuits has received sustained attention since the mid-twentieth century [1-4]. In such circuits, energy storage and energy dissipation play important role in many applications [5-9] and in particular in communications, and energy harvesting systems computing [10-13]. Moreover, there has been more interest in energy storage and energy dissipation with the emergence of Graphene and other nanomaterials with recent advances of supercapacitors [14-17]. Renewed interest in charging and discharging a capacitor in *R-C* circuit has been reported in the past several years [18-22]. Over the past decade, there has been much of interest in studying *R-C* circuit with gradual change of the charging process (adiabatic) [23-27] and in a previous study the present authors considered both theoretical and experimental stepwise charging a capacitor in RC circuit [28]. It is important to distinguish the present work on discharging from our previous study on charging [29]. Although charging and discharging appear symmetric, four key differences necessitate separate analysis. First, the initial conditions differ: charging starts from an uncharged capacitor (q=0), while discharging starts from a fully charged capacitor ($q=Q_0$). Second, the battery's role reverses: in charging, the battery supplies energy to the capacitor; in discharging, the battery receives energy from the capacitor. Third, the stability constraint differs: discharging requires the battery voltage to remain below the capacitor voltage at each step to maintain current opposition. Fourth, the applications differ: charging protocols optimize energy storage from a source, while discharging protocols optimize energy recovery from a stored charge. To our knowledge, closed-form expressions for stepwise discharge efficiency with experimental validation across a range of step numbers have not been previously reported.

There, the voltage of the power battery was increased through an arbitrary number of steps to achieve the final voltage. It was found that the energy loss in the resistor decreases as the step number increases. The conventional approach to discharging a capacitor employs a simple RC circuit, in which a capacitor of capacitance c, charge q_0 and voltage v_0 is connected to a resistor of resistance *R*. In such *R-C* circuit, all the initial stored energy in the capacitor is finally dissipated as heat energy in the resistor. The purpose of the present paper is to consider discharging a capacitor in *R-C* circuit by proposing adiabatic process, a technique which has been widely used in different areas [29-33]. In order to ensure the step-down nature of the voltage on the capacitor, we include in the *R-C* circuit a battery whose voltage, during any given step, is smaller than the initial voltage on the capacitor at the beginning of that step. In addition, the battery's own current must oppose the discharging current and that implies the positive terminal of the battery is connected to the positive plate of the capacitor. This ensures that

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the final voltage on the capacitor at the end of a given step equals to the voltage of the battery during that step. What fundamentally distinguishes this work from previous studies is that it provides the first complete theoretical derivation for stepwise discharging with a battery as an energy sink, supported by systematic experimental validation across N = 1, 2, 4, 8, 10, 20, 40, and the continuous ramp limit.

The organization of the paper is as follows: In section 2, we consider theoretical analysis for adiabatic discharging a capacitor, in which the heat dissipated energy and the energy gained by the battery are derived for any number of steps. In section 3, we construct an experiment through which numerical values for the heat dissipated energy and the gained energy by the battery are measured. In Section 4, we present theoretical and experimental analysis. In section 5, we provide results, discussion for our findings and practical limitations encountered in real circuits. Section 6 is devoted for conclusion.

Adiabatic discharging in RC circuit: Theoretical analysis

We consider *RC* circuit in which a charged capacitor of capacitance c connected in series to a resistor of resistance *R* and a battery of variable electromotive force ε. The aim is to discharge the capacitor (adiabatically) through an arbitrary number of steps *N.* This requires that, for each step, ε must be smaller than the voltage on the capacitor and the positive terminal of the battery is connected to the positive plate of the capacitor. This ensures that at the end of a given step, the capacitor undergoes partial discharge and the steady state (zero electric current) is achieved when the voltage on the capacitor becomes the same as that of the battery. We let the initial voltage and charge on the capacitor to be V_0 and Q_0 respectively. In order to make the analysis clear, we first examine the discharging during the first step in sec. (2.1) and then we consider discharging during the j^{th} step in sec. (2.2).

First-step Discharge

For the first step, the initial capacitor voltage is V_0 and the battery voltage is set to $\varepsilon = (N-1)V_0/N$. This means that the potential on the capacitor steps down by V_0/N by the end of this first step. Applying the Kirchhoff's loop rule, we get

$$\frac{q}{c} - R\frac{dq}{dt} = \varepsilon. \tag{1}$$

which is an inhomogeneous first-order differential equation whose solution is

$$q(t) = c\varepsilon + Ae^{-t/Rc}. \tag{2}$$

The constant *A* is determined by requiring that the initial charge on the capacitor is Q_0 at $t = 0$ and thus equation (2) becomes

$$q(t) = c\varepsilon(1 - e^{-t/Rc}) + Q_0 e^{-t/Rc} \tag{3}$$

The electric current can be found ($i = dq/dt$) as

$$i = \frac{1}{R}\left(\varepsilon - \frac{Q_0}{c}\right)e^{-t/Rc} \tag{4}$$

The decrease in the potential energy of the capacitor, the heat energy dissipated in the resistor and the energy gained by the battery during this step. Using the final voltage on the capacitor, $V = \varepsilon = (N-1)V_0/N$, the decrease in the potential energy on the capacitor is

$$|\Delta U| = U_i - U_f = \frac{1}{2}cV_0^2\left[1 - \left(\frac{N-1}{N}\right)^2\right] = \frac{1}{2}cV_0^2\frac{2N-1}{N^2}. \tag{5}$$

Using equation (4), the heat energy dissipated in the resistor is

$$W = \int_0^\infty i^2 R dt = \frac{1}{2}c\left(\varepsilon - \frac{Q_0}{c}\right)^2, \tag{6}$$

Substituting the value of $\varepsilon = (N-1)V_0/N$ and the initial value of the voltage on the capacitor $\frac{Q_0}{c}$, we get

$$W = \frac{1}{2}cV_0^2\frac{1}{N^2}. \tag{7}$$

This equation shows that the heat dissipated during each step scales as $\frac{1}{N^2}$, decreasing quadratically with the number of steps. In order to compute the energy gained by the battery, we have

$$E = V\Delta q = \frac{N-1}{N}V_0\Delta q, \tag{8}$$

whereΔq is the amount of charge leaves the capacitor and passes through battery,

$$\Delta q = c\Delta V = c\left(V_0 - \frac{N-1}{N}V_0\right) = c\frac{V_0}{N}. \tag{9}$$

Therefore, the energy gained by the battery is

$$E = cV_0^2\frac{N-1}{N^2}. \tag{10}$$

This energy gain can be verified by taking the difference between $|\Delta U|$ and W as shown from equations (5) and (7). Obviously, equations (7) and (10) show that the ratio between the gained energy and the heat energy dissipated, E/W, increases with *N* as $2(N-1)$.

General Step Analysis

Here we analyze discharging during the j^{th} step. The potential, Von the capacitor and the voltage on the battery are given by

$$V = \frac{N-j}{N}V_0 \tag{11}$$

$$\varepsilon = \frac{N-j-1}{N}V_0 \tag{12}$$

The differential equation that covers the discharging circuit is the same as equation (1), and therefore the form of the solution is similar to that of equation (2), namely

$$q(t) = c\varepsilon(1 - e^{-t/Rc}) + q_0 e^{-t/Rc}, \tag{13}$$

and therefore, the electric current is

$$i = \frac{dq}{dt} = \frac{1}{R}\left(\varepsilon - \frac{q_0}{c}\right)e^{-t/Rc} = -\frac{V_0}{RN}e^{-t/RC}, \tag{14}$$

where q_0 is the initial charge on the capacitor at the beginning of this step ($t = 0$),

$$q_0 = c\frac{N-j}{N}V_0, \tag{15}$$

and ε is given by equation (12). The magnitude of the energy change in the capacitor can be calculated using the initial potential as given by equation (11) and the final potential as given by equation (12), so we have

$$|U| = U_i - U_f = \frac{1}{2}cV_0^2\left[\left(\frac{N-i}{N}\right)^2 - \left(\frac{N-j-1}{N}\right)^2\right], \tag{16}$$

which could be simplified to

$$|\Delta U| = \frac{1}{2}cV_0^2\frac{(2N-2j-1)}{N^2}. \tag{17}$$

The heat energy dissipated in the resistor, using equation (14) is

$$W = \int_0^\infty i^2 R dt = \frac{1}{2}c\left(\varepsilon - \frac{q_0}{c}\right)^2. \tag{18}$$

The substitution for ε and q_0 from equations (12) and (15) allows us to write equation (18) as

$$W = \frac{1}{2}cV_0^2\left[\frac{N-j-1}{N} - \frac{(N-j)}{N}\right]^2 = \frac{1}{2}cV_0^2\frac{1}{N^2}. \tag{19}$$

Equation (19) shows that the dissipated heat energy is independent of the step number *j* and thus has the same value for each step.

Finally, the energy gained by the battery during the j^{th}step is calculated as follows:

$$E = V\Delta q = \left(\frac{N-j-1}{N}V_0\right)\Delta q, \tag{20}$$

whereΔq is given by

$$\Delta q = c(V_i - V_f) = c\left[\frac{N-j}{N}V_0 - \frac{(N-i-1)}{N}V_0\right] = c\frac{V_0}{N}. \quad (21)$$

Therefore, equation (20) becomes

$$E = \left(\frac{N-j-1}{N}V_0\right)c\frac{V_0}{N} = cV_0^2\frac{1}{N^2}(N-j-1), \quad (22)$$

This result equals the difference between $|\Delta U|$ and W as can be easily seen from equations (17) and (19).

Now by the end of the last step, we calculate the total dissipated heat energy, the magnitude of the energy change of the capacitor and the energy gained by the battery.

The total dissipated heat energy is easy to find, since the heat energy is the same for each step (see equation (19)) and therefore,

$$W_{tot} = \frac{1}{2}cV_0^2.\frac{1}{N^2}.N = \frac{1}{2}cV_0^2.\frac{1}{N}. \quad (23)$$

The above equation shows that the dissipated heat energy decreases as number of steps increases and it vanishes as $N \to \infty$.

The magnitude of the change in the energy of the capacitor is calculated using equation (17) and summing over all steps, $j$from 0 $\to (N-1)$, namely

$$|\Delta U|_{tot} = \frac{1}{2}cV_0^2.\frac{1}{N^2}\sum_{j=0}^{N-1}(2N-j-1) = \frac{1}{2}cV_0^2\frac{1}{N^2}\left[N(2N-1) - 2\frac{N(N-1)}{2}\right] = \frac{1}{2}cV_0^2. \quad (24)$$

This is expected since finally the capacitor will be totally discharged and hence all its initial energy will be given to the circuit.

The energy gained by the battery is also calculated by adding all energy gained in each step, therefore using equation (22) and summing over $j$from 0 $\to (N-1)$, we get

$$E_{tot} = cV_0^2\frac{1}{N^2}\sum_{j=0}^{N-1}\frac{1}{N^2}(N-j-1) = cV_0^2\frac{1}{N^2}\left[N(N-1) - \frac{N(N-1)}{2}\right] = \frac{1}{2}cV_0^2\frac{N-1}{N}. \quad (25)$$

Clearly the addition of this result and W_{tot} given in equation (23) gives exactly $|\Delta U|_{tot}$ given in equation (24). Equation (25) clearly shows that most of the energy stored in the capacitor will be gained by the battery, and in the limit of $N \to \infty$ the battery gains all the capacitor's stored energy $\left(\frac{1}{2}cV_0^2\right)$. For later computational purposes, we need to find the effective potential across the battery, V_{eff} , through which the charge will pass through. This could be found by requiring that the work done on the charge *q* when passing through this effective potential must equal to the energy gained by the battery, namely,

$$qV_{eff} = \frac{1}{2}cV_0^2\frac{N-1}{N}. \quad (26)$$

Using $q = cV_0$, equation (26) gives

$$V_{eff} = \frac{1}{2}V_0\frac{N-1}{N}. \quad (27)$$

The ratios of W_{tot} and E_{tot} relative to $|\Delta U|_{tot}$, can be obtained using equations (23), (24) and (25).

$$\frac{W_{tot}}{|\Delta U|_{tot}} = \frac{1}{N}. \quad (28)$$

$$\frac{E_{tot}}{|\Delta U|_{tot}} = \frac{N-1}{N}. \quad (29)$$

It is remarkable to note that we can store most of the capacitor's energy into the battery by increasing the step number *N* during discharging. For example, If a charged capacitor with potential V_0 is discharged through steps of $0.1\,V_0$ increment (i.e $N = 10$), then only 10% of the capacitor' energy will be dissipated while 90% will be gained by the battery. The limit $N \to \infty$ is equivalent to a stepdown ramp potential, and in this case all the capacitor's energy would be gained by the battery and there would be no dissipated energy. In order to explain this case, we derive below the current as function of time when the power source is a decreasing ramp potential. Consider the ramp potential $V(t) = V_0(1 - \alpha t/\tau)$, where $\tau\,(= Rc)$ is the time constant of the *RC* circuit and $\alpha$is a parameter so that the quantity $V_0\alpha/\tau$ determines the rate of the decrease of the potential V_0 down to zero. The smaller the value of $\alpha$the more adiabatic the discharging process will be. Applying Kirchhoff's law to the circuit gives us

$$\frac{q}{c} + iR = V_0\left(1 - \alpha\frac{t}{\tau}\right), \quad (30)$$

which can be written as

$$\frac{dq}{dt} + \frac{1}{Rc}q = \frac{V_0}{R}\left(1 - \alpha\frac{t}{\tau}\right). \quad (31)$$

The above equation is a linear, inhomogeneous, first-order differential equation, which has the same structure as the standard form, namely

$$\frac{dx}{dt} + p(t)x = f(t). \quad (32)$$

The well-known method to solve the above equation is by the integrating factor method. In this method, defining an integrating factor $u(t) = e^{\int p(t)dt}$, then the solution for Eq. (32) is

$$x(t) = \frac{1}{u(t)}\left(\int u(t)f(t)dt + A\right), \quad (33)$$

where *A* is a constant to be found from the initial condition. Applying this method to our differential equation (31), we get $u(t) = e^{t/Rc}$, $f(t) = (1 - \alpha t/\tau)V_0/R$, so that the integral in Eq. (33), gives (after integration by parts)

$$\int\frac{V_0}{R}(1 - \alpha t/\tau)e^{t/RC}dt = V_0\left(\frac{-\alpha t}{R} + 2C\right)e^{t/RC}. \quad (34)$$

Therefore, with $x(t) \to q(t)$, the solution of Eq. (30) is

$$q(t) = -\frac{V_0}{R}\alpha t + V_0C(1+\alpha) + Ae^{-t/RC}. \quad (35)$$

The constant *A* is found by requiring that the initial charge on the capacitor equalsCV_0, so that equation (35) gives $A = -CV_0\alpha$. Therefore, the charge is given by

$$q(t) = -\frac{V_0}{R}\alpha t + CV_0(1 + \alpha - \alpha e^{-t/RC}). \quad (36)$$

The electric current is now readily obtained using $i = dq/dt$ with the result

$$i = -\frac{V_0}{R}\alpha(1 - e^{-t/Rc}). \quad (37)$$

Therefore, after a long time $t \gg \tau$, the electric current attains a constant value, $-V_0\alpha/R$. This explains the plateau behavior in Fig. (3). It is remarkable to calculate the dissipated heat energy, using equation (37), we have

$$W_{theo} = \int_0^{\tau/\alpha} i^2\,R\,dt = \frac{(V_0\alpha)^2}{R}\int_0^{\tau/\alpha}(1 - e^{-t/RC})^2\,dt = V_0^2\alpha^2C\left(\frac{1}{\alpha} - \frac{3}{2}\right) \cong V_0^2\alpha C. \quad (38)$$

Note that for a meaningful adiabatic process, the magnitude of the slope of decreasing ramp potential must be very small and therefore, $\ll 1$. It will be seen later, when we discuss the experimental part, that $\alpha = 5.53 \times 10^{-3}$. This justifies the approximation made in the last step of equation (38). Furthermore, we calculate the gained energy for this case: For a time $t = \alpha/\tau$the charge on the capacitor is given by equation (36) which gives $CV_0\alpha$. Therefore, the charge that passes through the battery by that time is $CV_0 - CV_0\alpha = CV_0(1-\alpha)$. The energy gained by the battery is thus

$$E_{theo} = qV_{ave} = \frac{1}{2}CV_0^2(1-\alpha). \quad (39)$$

Experimental analysis

Charging and discharging have been performed to RC circuit using controlled PGZ 402 potentiostate (radiometer analytical). *RC* circuit was connected using two electrode connections mode as shown in Fig. (1). Charging the capacitor to a final potential

V_0 was conducted using one step potential of V_0, while discharging was performed using N steps potential (N = 1 to infinity) that decreases from V_0 to zero.

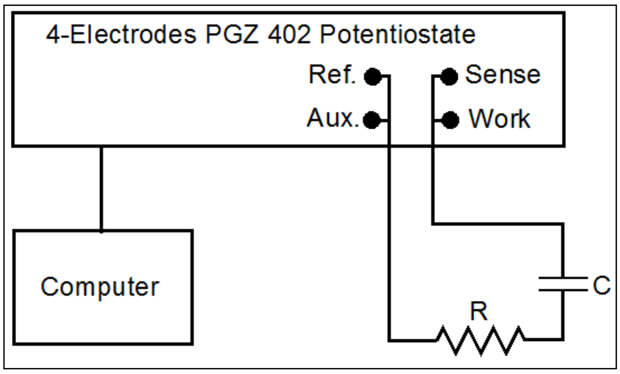

Figure (1): Schematic for RC circuit connected to PGZ 402 Potentiostate using two electrode mode connection.

Prior to discharging process, we first charge the capacitor by one- step potential of $2\ V$ to get to its maximum charge of to its maximum charge as shown in Fig. 2. Charging continues until the current reaches zero, which means the charge approaches its final value. The value of the final charge is the area under the current-time curve and is found to be $941\ \mu C$.This value is used to find the capacitance of the capacitor by using $c = q/V_0$with $V_0 = 2\ V$, we get $c = 470.5\mu F$. The value of the resistance used in the experiment is 4700Ω.

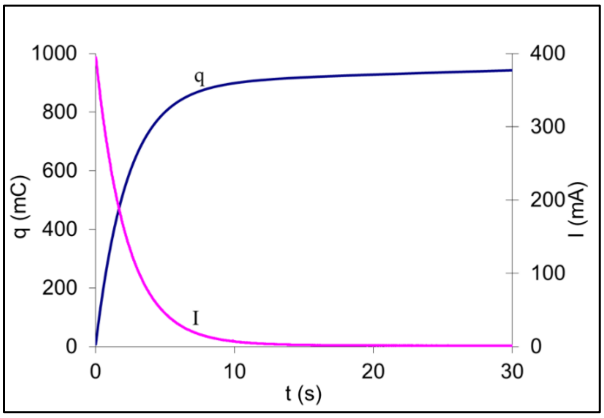

Figure (2): Evolution of charge and current with time during charging using one step potential of 2V.

The procedure for adiabatic discharging the capacitor is performed by using decreasing *N* steps potential. This means that for a given step number *N,* the capacitor starts discharging at potential $V_0 = 2V$ and decreases by V_0/N for each step. This implies that, for the j^{th} step, the capacitor starts discharging at a potential $V_0(N-j)/N$ and ends at $V_0(N-j-1)/N$ when the current reaches zero. Finally, after *N* steps, the capacitor is totally discharged and current-time curves are plotted for some selected values of *N* which are shown in Fig. 3.

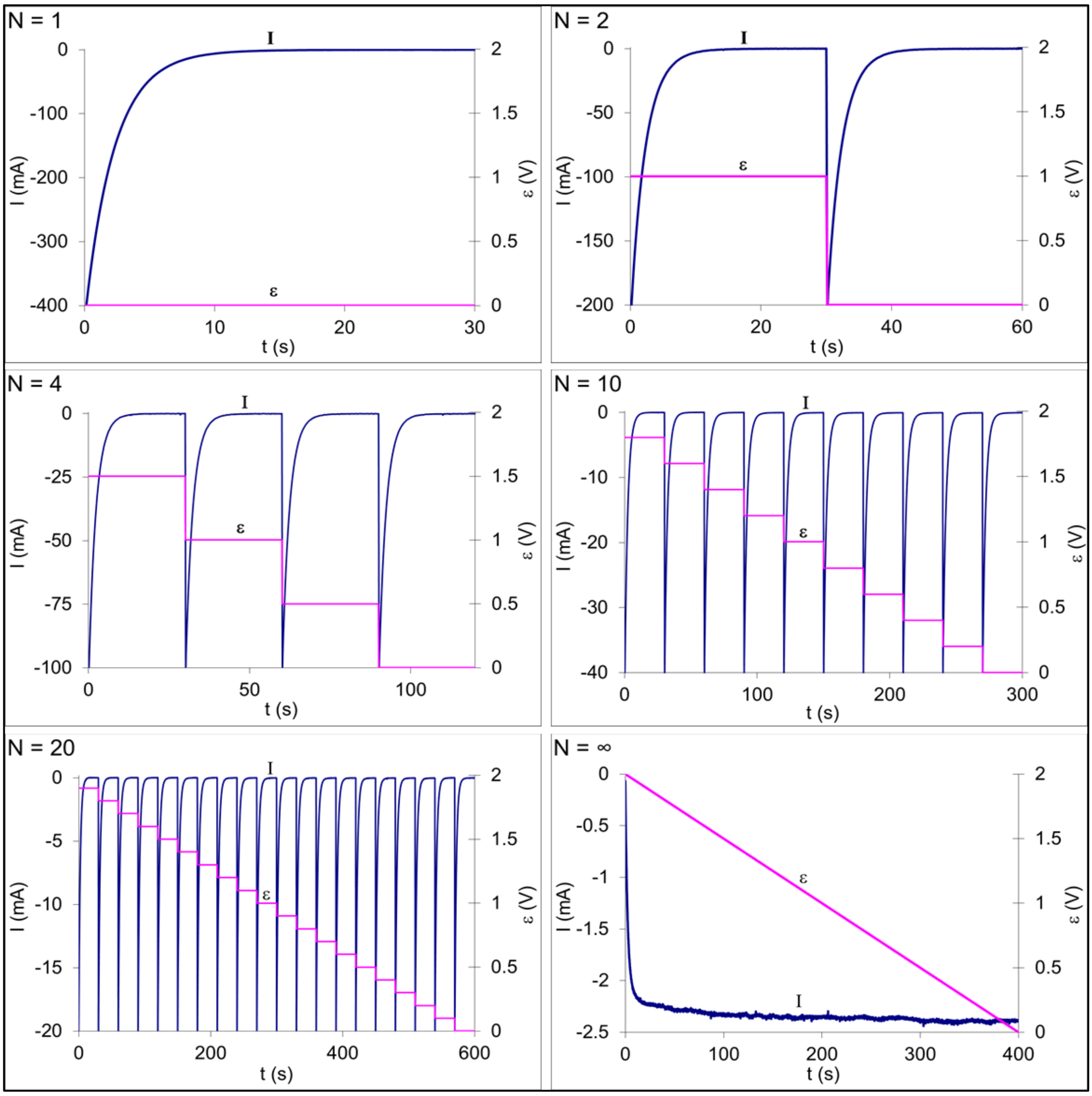

Figure (3): Current vs. time during discharging capacitor by different N step potential ε (N = 1, 2, 4, 10, 20, ∞).

Theoretical and experimental calculations

The value of the capacitance of the capacitor is $470.5\ \mu F$, the potential $V_0 = 2V$ and the resistance $R = 4700\Omega$. For the theoretical values: The initial energy stored in the capacitor, U_{theo}, is given by equation (24), the gained energy by the battery, E_{theo}, is given by equation (25) and the dissipated heat energy, W_{theo} is given by equation (23). For the experimental values: The energy stored in the capacitor is $U_{exp} = q^2/2c$, the gained energy by the battery, $E_{exp} = qV_{eff}$, with V_{eff} is given by equation (27), and the dissipated heat energy, W_{exp} is the difference between U_{exp} and E_{exp}.

So for each step number *N,* the charge is measured from the area under the current-time curve and the above quantities are calculated and shown in table 1. Experimental uncertainties were estimated as follows. The charge q was obtained by numerical integration of the current-time curve using the trapezoidal rule; the integration uncertainty of ±3 μC was determined from the digitization resolution (0.1 μA) and the sampling interval (1 ms). For each N, three independent discharge measurements were performed. Propagated uncertainties for U_{exp}, E_{exp}, and W_{exp} were calculated using standard error propagation formulas, assuming uncorrelated errors in q and V_{eff}. Instrumental uncertainties include the potentiostat voltage accuracy (±0.5%) and current resolution (±0.1 μA)

Table (1): Theoretical and experimental values for stored energy, U, gained energy, E, and heat dissipated energy, W, for selected values for the step number *N*.

N	q (μC)	V_{eff} (V)	$U_{theo.}$(μJ)	$E_{theo.}$(μJ)	$W_{theo.}$(μJ)	$U_{exp.}$(μJ)	$E_{exp.}$(μJ)	$W_{exp.}$(μJ)
1	926	0.000	941	0.00	941	911	0.00	911
2	932	0.500	941	471	471	923	466	457
4	939	0.750	941	706	235	937	704	233
8	940	0.875	941	823	118	939	823	117
10	942	0.900	941	847	94.1	943	848	95.2
20	943	0.950	941	894	47.1	945	896	49.2
40	942	0.975	941	917	23.5	943	918	24.6
∞	933	1.00	941	935.8	10.2	925	933	7.93

* All experimental values have an estimated uncertainty of ±3%. Charge measurements have an uncertainty of ±5 μC.

Results and discussion

The behavior of the graphs shown in Fig. 3 agrees with the theoretical predictions with our theoretical results: For any step number N , the current, at $t = 0$, has its maximum negative value which is given by $-V_0/NR$ from equation (14). Using $V_0 = 2V$and $= 4700\Omega$, verification shows this agreement for each value of N. For the decreasing ramp potential, the current is zero at $t = 0$ as seen from equation (37) which is verified experimentally as seen from the graph for that case in Fig. (3). Furthermore, the slope for the ramp potential is $5 \times 10^{-3} V/s$, so equating this value with $V_0\alpha/\tau$, with $\tau = RC$, one finds $\alpha = 5.53 \times 10^{-3}$. Therefore, after a very long time, the current $i = -\frac{V_0\alpha}{R} = -2.35\mu A$, which is close to the experimental value $-2.5\mu A$.

Table 1 shows the theoretical and experimental values for the stored energy U, the gained energy, E and the dissipated heat energy, W for some selected values of the step number, N. It is noticed a good agreement between the theoretical and experimental values. For the decreasing ramp potential case: The theoretical value of the dissipated heat energy is $10.2\mu J$ as calculated from equation (38) which agrees with the experimental value within measurement limitations $7.93\mu J$. The theoretical value of the energy gained by the battery, using equation (39), is $935.8\,\mu J$, which is close to its corresponding experimental value ($933\,\mu J$) as seen in table 1. It should be noted that as α is made smaller (i.e discharging process is more adiabatic), the dissipated heat energy gets smaller and the gained energy gets larger. In the limit $\alpha \to 0$, the dissipated heat energy vanishes and the all the initial stored energy in the capacitor will be gained by the battery, as seen by equations (38) and (39) respectively. For comparison purposes, we calculate the ratios of energy gained and dissipated heat energy relative to the energy stored in the capacitor for the selected values of the step number, this is shown in table 2. We note an excellent agreement between the theoretical values and the experimental values of these ratios.

Table (2): Ratios of gained energy and dissipated energy relative to stored energy for some step number values.

N	$(E/U)_{theo.}$	$(W/U)_{theo.}$	$(E/U)_{exp.}$	$(W/U)_{exp}$
1	0.00	1.00	0.00	1.00
2	0.50	0.50	0.50	0.50
4	0.75	0.25	0.75	0.25
8	0.88	0.13	0.88	0.12
10	0.90	0.10	0.90	0.10
20	0.95	0.05	0.95	0.05
40	0.98	0.03	0.97	0.03
∞	0.99	0.01	1.01	0.01

Real circuits limitations

The performance of adiabatic capacitor discharging is influenced by several non-ideal practical limitations. First, the real battery internal resistance and the resistance of connecting wires introduce energy dissipation, which is not accounted for the ideal model. Second, generation of voltage fluctuations due to thermal (Johnson) noise, which can become comparable to the step size at higher resolution (high N), reducing measurement precision. The time setting of the potentiostate must be sufficiently smaller than the RC time constant, especially for faster circuits with smaller time constants, which may limit the maximum achievable N. Fourth, since electrolytic capacitor is used in this experiment, its leakage current and temperature variations can introduce a significant error. Taking all these factors into consideration, practical adiabatic discharging is limited to N = 40, to ensure efficiency higher than 90%.

Conclusion

In this work, theoretical and experimental stepwise (adiabatic) discharging in *RC* circuit has been investigated. Analytical formulas for the dissipated heat energy and gained energy by the battery had been derived. It has been shown that as the step number increases, the dissipated heat energy decreases and the gained energy by the battery increases. An interesting limiting case is that when the step number goes to infinity, in which the dissipated heat energy vanishes and the battery would gain all the initial stored energy in the capacitor. This case was modeled by a decreasing ramp potential whose slope has a magnitude much smaller than one. The other part of this work is an experimental setup in which the dissipated heat energy and the gained energy have been measured experimentally. Both of our theoretical and experimental results are in a very good agreement. This work demonstrates a method for storing energy by using adiabatic process for discharging a capacitor. This work is the first to provide both complete theoretical expressions for stepwise discharging energy recovery and systematic experimental validation across a wide range of step numbers including the continuous ramp limit. Extending this method to inductive circuits, AC applications, or integration with switched-capacitor converters remains unexplored and presents opportunities for further research.

Disclosure Statement

- **Ethics Approval and Consent to Participate:** Not applicable. This study involves no human participants, animals, or biological materials; it is purely a theoretical and experimental physics investigation using electronic components.
- **Consent for Publication:** Not applicable. The manuscript contains no individual person's data, images, videos, or case reports.
- **Availability of Data and Materials:** The raw data required to reproduce these findings are available in the body and illustrations of this manuscript.
- **Author's Contribution:** The authors confirm contribution to the paper as follows: study conception and design: Al-Jaber SM; theoretical calculations, derivations, and modeling: Saadeddin I; experimental setup, data collection, and validation: Saadeddin I, Al-Jaber SM; data analysis and interpretation: Saadeddin I, Al-Jaber SM; draft manuscript preparation: Saadeddin I, Al-Jaber SM. All authors reviewed the results and approved the final version of the manuscript.
- **Funding:** No funding was received for this work.
- **Conflicts of Interest:** The authors declare that there is no conflict of interest regarding the publication of this article.
- **Acknowledgements:** The authors thank the Department of Physics at An-Najah National University for providing laboratory facilities and equipment used in this study.

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