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Bedload Transport in Gravel Streams - Surface Versus Subsurface Based Transport Analysis

انتقال الاتربة في قيعان الانهار – التحليل وفق التدرج الحبيبي للطبقات السطحية بدلاً من الطبقات تحت السطحية

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Abstract:

Fluid forces and the quantity and ranges of grain sizes present on a streambed are the important factors affecting the transport of sediments. Most bedload transport models are based on single grain size parameter that represents either the surface or the subsurface bed material. Fractional bedload transport analysis is used to account for the different grain sizes. Surface versus subsurface-based fractional analysis of available bedload transport data are compared and the effect of the selection of the dimensionless reference bedload transport parameter is investigated. Fractional analysis has indicated that it is referenced to the size distribution of the bed surface rather than the subsurface and that its necessity increases for bedload data sets that fall closer to the lower end near threshold conditions. The reference value for the dimensionless bedload parameter used in this paper falls within the lower range of the analyzed bedload data set.

ملخص:

القوى الهيدروليكية إضافة إلى الكميات والتدرج الحبيبي للأتربة الموجودة في قيعان الأنهار هي أهم العوامل التي تؤثر على انتقال هذه الأتربة. معظم النماذج التي تتعامل مع انتقال الأتربة تعتمد على تدرج حبيبي ممثلاً للأتربة والمواد في الطبقات السطحية أو الطبقات تحت السطحية لقيعان الأنهار. ولكي يتم أخذ التدرج الحبيبي بعين الإعتبار يجب تحليل انتقال الأتربة في قيعان الأنهار وحساب حمل القاع (Bed load) بوساطة نماذج تعتمد على التحليل الجزئي للتدرج الحبيبي. في هذه الورقة تم مقارنة النماذج التي تعتمد التدرج الحبيبي

للطبقات السطحية مع تلك التي تعتمد التدرج الحبيبي للطبقات تحت السطحية وذلك لحساب انتقال الأتربة وحمل القاع (Bed load) وكذلك دراسة تأثير اختلاف القيمة المرجعية (W^*_r) لانتقال الأتربة في قيعان الأنهار. تبين من الدراسة أن التحليل الجزئي (Fractional Analysis) يجب أن يستند إلى التدرج الحبيبي للطبقات السطحية بدلاً من الطبقات تحت السطحية وأن أهمية هذا التحليل وضرورته تزداد كلما كانت النتائج قريبة من القيم التي تبدأ عندها الحركة من حيث التدرج والقوى الهيدروليكية، فقد أثبتت الورقة أن القيمة المرجعية لحساب انتقـال الأتربة في قيعان الأنهار(W^*_r) يجب أن تكون في حدود القيم الدنيا لمعطيات انتقال الأتربة والمـواد المُـراد تحليلها.

INTRODUCTION

The transport of sediments from a streambed of mixed sizes depends on the availability of each grain size present on the bed surface and the fluid forces on the exposed grains. In a gravel-bed stream, the bed material is often sorted such that the surface composition is coarser than the subsurface. Many mixed size transport formulas have been developed relative to the grain size of the subsurface material rather than the bed surface. For low Shields stresses the surface layer (pavement) is the main contributor to bedload transport. For higher stresses the subsurface layer is exposed and also contributing to the bedload and, eventually, equal mobility conditions govern. As to Parker(1), substrate particles can participate in the bedload only to the extent that the local or global scour results in their exposure on the surface.

To accurately estimate sediment transport loads and the participation of the surface versus subsurface particles in the bedload, it may be necessary to know the continuous change of the grain size composition of the surface layer with flow conditions. This is typically unknown, especially during flood conditions, when most of the material is transported. During floods and high shear stress, most bed material is exposed to transport and conditions of equal mobility prevail.

Almedeij & Diplas(2) have used a two-parameter approach, surface and subsurface combination, for predicting bedload transport rates in gravel bed streams. Their formula uses two particle size diameters, one to represent the surface and the other the subsurface materials. The formula implicitly accounts for the variation in the make-up of the surface bed material over a wide range of Shields stresses. This approach is believed to reflect the resulting changes in the contributions made by the pavement (surface) and sub-pavement layers to the bedload transport.

Fractional bedload transport analysis consists of dividing the bed material distribution into several size ranges, each represented by a particle diameter, D_i. Fractional analysis requires that the bed size distribution be specified for scaling purposes, however, either the surface or the subsurface size distribution may be used. Traditionally, the subsurface size distribution has been used since it is a known stable distribution that does not vary significantly with flow conditions. Researchers such as Parker(1), Wilcock & McArdell(3) and Wilcock & Crowe(4) have proposed surface based fractional bedload transport approaches. Parker et al.(5) and Diplas(6) have implemented subsurface-based fractional bedload transport rate calculations for poorly sorted sediments.

Here surface versus subsurface-based fractional analysis of available bedload transport data is presented and compared. The effect of the selection of the dimensionless reference bedload transport parameter, W_i^*, on the similarity collapse and fractional analysis is also investigated.

EXPERIMENTAL DATA

The experimental data obtained by Proffitt(7) are suitable for investigating a wide range of transport rates. He conducted experiments in a non feeding, non-recirculating sediment flume to study armouring due to transport of nonuniform sediments. The bed material and bedload size distributions, bedload transport rates, and corresponding hydraulic data were measured and are suitable for fractional transport analysis using a surface-subsurface combination model. The laboratory data of Proffitt(7) is used here in the fractional analysis.

Proffitt performed four series of experiments. Each series consisted of four experiments. The bed material was kept constant for each series, while each experiment was designed with a different shear stress. Three phases could be identified within each experiment. The initial phase, which typically lasted for about one hour, was characterized by intense and relatively constant bedload transport, measured at the downstream end of the flume. In the initial phase, both surface and subsurface particles (bulk size distribution) have been exposed to transport. The final phase was characterized by the coarsest pavement, which was distinct for each of the sixteen experiments, a condition reached after 20 to 95 hours of run time, and a bedload transport rate of 2.5% or lower of that measured during the initial phase of the corresponding experiment. During the intermediate phase, the channel bed and bedload transport rate transitioned from the initial to the final phase conditions.

Proffitt used a trap with a settling basin to collect the transported sediment at the downstream end of the flume. It is likely that some particles moving in suspension could have settled and were subsequently included in the bedload measurements. In addition, it is noted that fine material, comprising between 0.2 and 1.7% of the total sediment transported during a run, was carried over the top of the trap. The weights and grain size distributions were then adjusted in proportion to the total amount collected. Thus it is necessary to determine the size ranges carried in suspension and remove them from the fractional bedload analysis.

Using the Bridge & Bennett[8] suspension criterion, for sediments having a submerged specific gravity of 1.65 mm and an assumed shape factor of 0.7, it is determined that the maximum particle size in suspension for both the initial and final phases of Proffitt's experiment ranged from 0.3 to 0.5 mm. Therefore, the present analysis excluded particles smaller than 0.5 mm and dealt with the portion of sediment transported as bedload. Sediments with D_i < 0.5 mm, constituted a percentage between 0.21 and 5.31 of the material collected in the sediment trap in all the experimental runs.

FRACTIONAL ANALYSIS

Several authors employed the fractional analysis approach; among them are Parker et al.[5] and Diplas[6]. In the approach, the bed material of both surface and subsurface size distributions is divided into ten grain size ranges, starting with D_i = 0.70 mm and ending with D_i = 15.55 mm, where D_i represents the geometric mean diameter of the *i*th grain size range. The dimensionless bedload parameter for each size range, W_i^*, is correlated with the corresponding Shields stress, τ_i^*, with

$$W_i^* = \frac{q_{B_i}^*}{\left(\tau_i^*\right)^{1.5}} = \frac{Rq_{B_i}}{f_i\left(dS\right)^{1.5}\sqrt{g}} \quad (1)$$

$$\tau_i^* = \frac{\tau_o}{\rho gRD_i} \quad (2)$$

$$q_{B_i}^* = \frac{q_{B_i}}{f_i D_i\sqrt{RgD_i}} \quad (3)$$

$$\tau_o = \rho gdS \quad (4)$$

where q_{B_i} denotes the volumetric bedload transport rate of the *i*th grain size range per unit channel width; $q_{B_i}^*$ denotes the Einstein bedload parameter for the *i*th grain size range; f_i denotes the fraction of the surface or subsurface material represented by D_i; ρ denotes the fluid density; *g*, the acceleration gravity; *R*, the submerged specific gravity of sediments; τ_o, the boundary shear stress; *d*, the flow depth; and *S*, the slope of the energy line.

Figure 1 plots the $W_i^* - \tau_i^*$ relation for the ten size ranges of the initial phase and final phase bedload transport sediments. The subsurface sediment gradation (four different series) has been used for the analysis of the initial phase data (subsurface based). For the final phase data, the plots represent the results based on the sediment gradation of the coarsest pavement, which was distinct for each of the sixteen experiments (surface based).

Tables 1 and 2 show the grain size ranges and the mean grain diameter of each range used to divide the experimental data. Table 2 compares the results of surface based versus subsurface-based fractions analysis of final phase bedload data. The tables list the results of the log-linear regression of the $W_i^* - \tau_i^*$ relation of the form

$$W_i^* = \alpha_i \left(\tau_i^*\right)^{m_i} \qquad (5)$$

for both the initial phase (Table 1) and the final phase (Table 2) based analyses. Further analysis of the final phase bedload transport data is conducted to investigate the effect of relating the bedload transport to the grain size distribution of the surface versus that of the subsurface.

The obtained m_i values for the initial phase (subsurface based) range from 1.90 to 3.87. For the final phase the values range from 1.28 to 5.05 for the surface based analysis and from 0.63 to 5.69 for the subsurface based analysis. The exponent, m_i, for the initial phase data showed rather limited variation and no defined trend, implying that most of the grain sizes, especially for the first eight, were transported under equal mobility conditions during the initial phase of the experiments. A value of 2.0 accurately reflects the trend exhibited by the first eight size ranges of the initial phase bedload data (Fig. 2). The last two size ranges of the initial phase have an average exponent value m_i of about 3.78.

For the final phase bedload data of the experiments, a small but consistent increase in the value of the exponent, m_i, with grain size is exhibited by the data. The rate of increase is higher for the surface-based

(1.73) than that for the subsurface-based analysis (1.10) indicating a higher response of the surface material to the increase in the shear stresses. This can be explained as due to the physical phenomenon of pavement that characterizes gravel-bed streams. The exposure and thus the response of the surface material to an increase in the shear stresses are higher than that of the subsurface material. On the other hand, the exponent m_i of the final phase bedload data is lower for the subsurface-based analysis than for the surface-based. Both surface and subsurface-based analyses have indicated that the data exhibit the same trend of consistent increase in the value of the exponent, m_i, with grain size (Fig. 2). D_i = 0.72 showed an exception from the increasing trend of the rate. Proffitt[7] indicated that particles smaller than 1.0 mm were frequently in suspension during the experiment and that D_i = 0.72 could be excluded from the analysis and considered as suspended load. This explains the above exception.

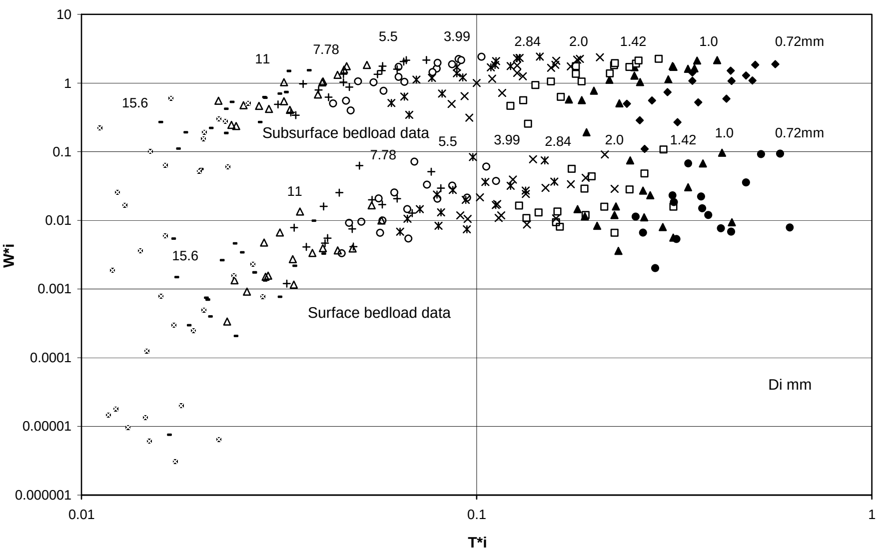

Figure 1: Plot of W_i^* versus τ_i^* for the ten size ranges of initial (subsurface) and final phase (surface) bedload data

On the upper part of τ^*, the subsurface-based resulted in lower values of W^*, whereas on the lower side, higher values of W^* were obtained. This is because the subsurface distribution provides higher fraction ($\% f_{iav}$) for the smaller grain size ranges and lower fraction for the larger grain size ranges than the surface distribution due to the pavement phenomenon and the characteristics of pavement layer. The $\% f_{iav}$ in tables 1 and 2 is average of the four series for the subsurface-based and of the sixteen series for the surface-based.

Table 1: Regression results of $W_i^* = \alpha_i \left(\tau_i^*\right)^{m_i}$ for initial phase bedload data

Size Range (mm)	D_i (mm) (1)	m_i (2)	α_i (3)	r^2 (4)	$\% f_{iav}$ (5)
0.6 – 0.853	0.72	2.21	6.65	0.57	3.91
0.853 - 1.20	1.01	1.97	14.66	0.66	7.13
1.20 - 1.68	1.42	2.02	33.72	0.75	10.01
1.68 - 2.41	2	1.96	64.83	0.77	13.67
2.41 - 3.35	2.84	1.93	121.86	0.78	11.05
3.35 - 4.76	3.99	1.90	208.74	0.84	16.00
4.76 - 6.35	5.5	2.06	528.51	0.84	16.50
6.35 - 9.52	7.78	2.14	975.28	0.77	5.41
9.52 - 12.7	11	3.87	5.14×10^5	0.55	4.54
12.7 - 19.0	15.55	3.69	2.1×10^5	0.32	4.61

Table 2: Surface versus subsurface-based regression results of $W_i^* = \alpha_i \left(\tau_i^*\right)^{m_i}$ for final phase bedload data

Size Range (mm)	D_i (mm)	Surface Based Analysis				Subsurface Based Analysis			
		m_i	α_i	r^2	$\% f_{iav}$	m_i	α_i	r^2	$\% f_{iav}$
0.6 – 0.85 0.85 -	0.7 2 1.0	1.7 8 1.2	0.087 0.090 0.181	0.20 0.14 0.22	2.07 4.29 4.82	1.19 0.63 0.80	0.03 0.02 0.04	0.1 3 0.0	3.91 7.13 10.0

1.20 1.20 - 1.68 1.68 - 2.41 2.41 - 3.35 3.35 - 4.76 4.76 - 6.35 6.35 - 9.52 9.52 - 12.7 12.7 - 19.0	1 1.4 2 2 2.8 4 3.9 9 5.5 7.7 8 11 15. 55	8 1.3 5 1.6 7 1.8 6 2.2 8 2.6 5 2.8 3 3.8 1 5.0 5	0.612 1.634 4 7.807 33.05 39.17 1.3x1 0^{3} 5.5x1 0^{4}	0.38 0.45 0.51 0.48 0.55 0.39 0.38	6.58 5.23 13.9 4 17.6 4 7.89 12.6 8 16.2 7	0.96 1.23 1.36 1.86 2.30 4.10 5.69	0.08 0.22 0.47 2.66 12.60 1.0x1 0^{4} 2.5x1 0^{6}	4 0.0 8 0.1 7 0.2 8 0.2 7 0.3 6 0.4 3 0.3 6 0.4 0	1 13.6 7 11.0 5 16.0 0 16.5 0 5.41 4.54 4.61

Figure 2 plots the relation between m_i and D_i/D_{50} for both initial and final phases of the bedload transport data and compares between the surface and subsurface-based analysis of the final phase bedload data. D_{50} equals 3.35 as an average of the four series of the subsurface-based and 7.22 as the average of the sixteen series of the surface-based analysis. The above analysis indicates that fractional analysis is more correctly referenced to the size distribution of the bed surface rather than the subsurface for the final phase data. For the initial phase data, fractional analysis can be conducted using only two size ranges instead of ten, these are 0.72 - 9.52 and 9.52 - 19.00 represented by the two geometric means 2.62 and 13.45 mm. The resulting exponents, m_i, are then 1.95 and 3.58 respectively. The first is about the same as the average of the first eight size ranges of the initial phase, expressing equal mobility conditions (Table 1), whereas the other exponent, 3.58, is little less than those of the last two size ranges.

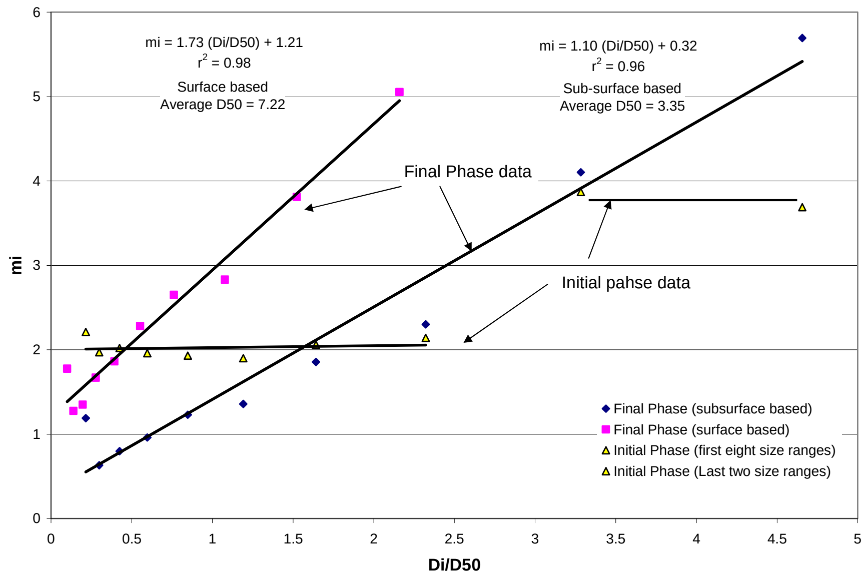

Figure 2: Plot of the exponent m_i versus D_i/D_{50} of both initial and final phase bedload data

SIMILARITY COLLAPSE

The similarity collapse approach is applied to represent the entire $W_i^* - \tau_i^*$ relations for the different grain sizes of each phase by one form. The approach has been applied by different literatures. Parker et al.[5] and Diplas[6] have reduced the $W_i^* - \tau_i^*$ curve for the different grain sizes to a single curve of the form:

$$W_i^* / W_r^* = \left(\tau_i^* / \tau_{ri}^*\right)^{ni} = \Phi_i^{ni} \qquad (6)$$

where W_r^* is a reference value for the dimensionless bedload parameter; τ_{ri}^* is the Shields stress capable of transporting bedload $W_i^* = W_r^*$; $\Phi_i = \tau_i^* / \tau_{ri}^*$ is the normalized Shields stress. Values of τ_{ri}^* are determined using the regression equation of fractional transport rate of each grain size and are sometimes estimated by eye on plots for each fraction.

Most of the literatures have used a reference value of W_r^* = 0.0025, which is thought to represent conditions slightly above threshold of motion. For Proffitt's data, the reference dimensionless bedload parameter, W_r^*= 0.0025, would be less than all W_r^* values of the initial phase (subsurface bedload data). Only one value of the grain size D_i=15.6 mm (W_r^*= 0.0019) is less than 0.0025 (Fig. 1), whereas W_r^* = 0.0025 falls within several points of the final phase (surface bedload data). To evaluate the effect of the selection of the reference dimensionless bedload parameter on the similarity collapse approach, different values for W_r^* are selected and the similarity collapse is applied for both initial and final phase bedload data. Table 3 summarizes the results of the regression applied to equation 6 for different values of the bedload parameter. Examining Table 3, it can be concluded that for the initial phase data a value of W_r^* between 0.1 and 1 gives the average exponent (n_i) and better regression coefficient. For the final phase data the value W_r^* = 0.0025 gives a value (n_i = 1.84) that about equals the average exponent and results better regression coefficient. Both the reference W_r^*, 0.1 and 0.0025 values suggested here as most appropriate for collapsing the data, cross the corresponding bedload, initial and final, at their lower range of values.

Table 3: Similarity Collapse results of bedload transport data for different reference values W_r^*

	Initial Phase bedload data					Final Phase bedload data			
W_r^*	0.0025	0.001	0.01	0.1	1.0	0.0025	0.001	0.01	0.1
n_i	2.07	2.07	2.05	2.04	3.00	1.84	1.70	3.17	2.62
r^2	0.26	0.04	0.53	0.73	0.71	0.63	0.65	0.65	0.30

Parker et al.[5] used weighted average exponent for the different grain size ranges. The weighted average was calculated based on the fraction of the bed material, which was assumed not changing with flow conditions. Here the fraction is changing with the flow conditions and Shields stress parameter, which reflects the variation of the flow and bed material conditions in natural streams. Using the average fraction, $\% f_i$ calculated in column (5) of Table 1 and that of table 2, the average exponent for the ten grain size ranges of the initial phase bedload data is 2.17 and that of the final phase is 2.93.

The above indicates that the similarity collapse technique is sensitive to the selection of the reference dimensionless bedload parameter and that W_r^* should be selected to vary as to the Shields stress parameter and flow conditions. The exposure and response of bed material to the applied shear stresses and thus to bedload transport is varying and subject to change with flow conditions. Therefore the reference bedload parameters should not be constant and should vary accordingly.

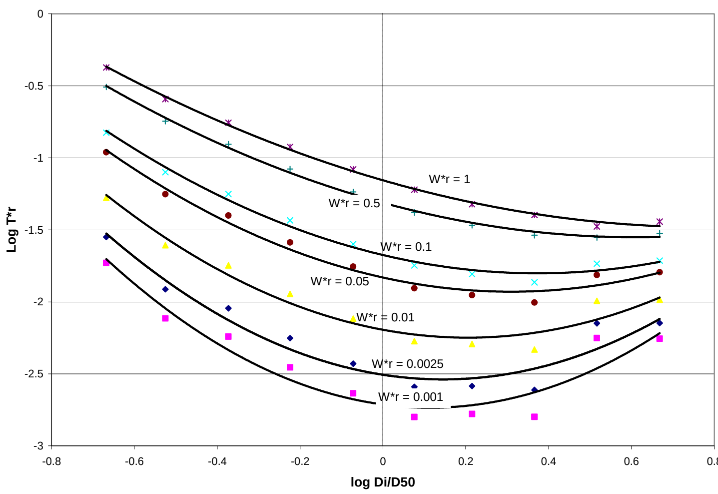

Figure 3: Variation of reference Shears stress τ_{ri}^* versus D_i/D_{50} for different W_r^* values for similarity collapse of initial bedload transport data

Figures 3 and 4 plot the relation between τ_{ri}^* and D_i/D_{50} for different W_r^* for initial and final phase bedload data respectively. The τ_{r50}^* values for different W_r^* are presented by the intersection of the different plots with the vertical axis passing the value log D_i/D_{50} = 0. The figures present also intermediate values 0.05 and 0.5 for W_r^*.

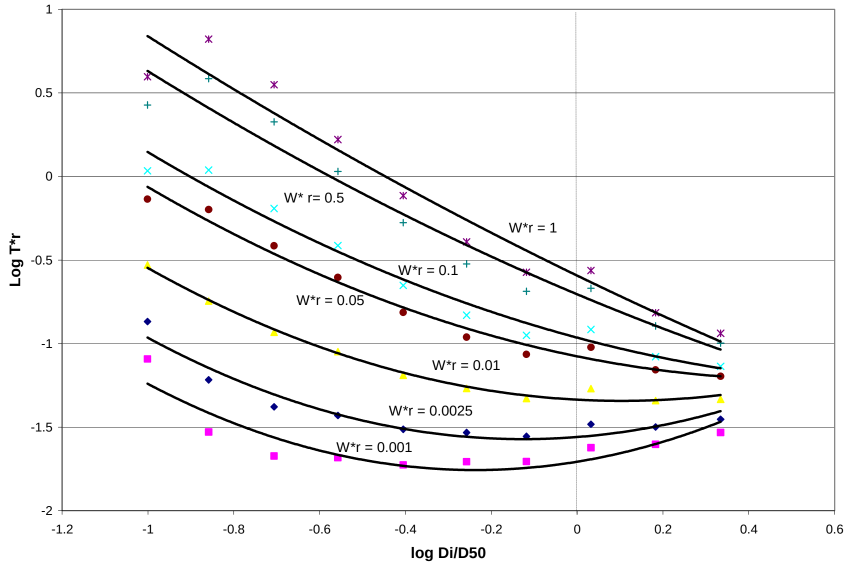

Figure 4: Variation of reference Shears stress τ_{ri}^* versus D_i/D_{50} for different W_r^* values for similarity collapse of final bedload transport data

EQUAL VERSUS NON-EQUAL MOBILITY CONDITIONS

As to Parker[1] and Parker et al.[5], bedload transport in gravel streams is accomplished by means of the mobilization of grains exposed on the bed surface. The mobilization results from the action of fluid forces on the exposed grains. A consequence of the equal mobility hypothesis is that bed-load size distribution is approximated by that of the substrate for all flows capable of mobilizing most available gravel sizes[9]. This is the strong form of the concept of equal mobility as defined by Parker & Toro-Escobar[10]. The initial phase of Proffitt data represents conditions of equal mobility. For final phase data, the grain sizes are paved and the non-equal mobility conditions prevail.

One way to check the equal versus non-equal mobility conditions of both surface and subsurface sediments is to plot the relation between the reference Shield stress τ^*_{ri} and D_i/D_{50} ratio. The form of the equation is[12]

$$\tau^*_{ri} = \tau^*_{r50}\left(D_i/D_{50}\right)^{\beta} \qquad (9)$$

where τ^*_{r50}, is the reference Shields stress associated with the D_{50}, the median grain size of the material. The exponent β = -1 indicates equal mobility conditions. For β > -1 finer material is more mobile than coarse material and non-equal mobility conditions prevail. For this case the pavement phenomenon is not encountered. For β < -1 the coarser particle become more mobile again non-equal conditions prevail. Here pavement and hiding conditions prevail.

Figure 5 plots the relation between Shield stress τ^*_{ri} and D_i/D_{50} ratio for both initial and final phases of Proffitt bedload data for two different values of W^*_r. The figure did not plot a linear relation on a log-log scale as expected. The D_i/D_{50} values, which are greater than one, did not follow the trend of the linear relation as for the values D_i/D_{50} less than one. This might be due to the averaging of the D_{50} for the different grain size distributions.

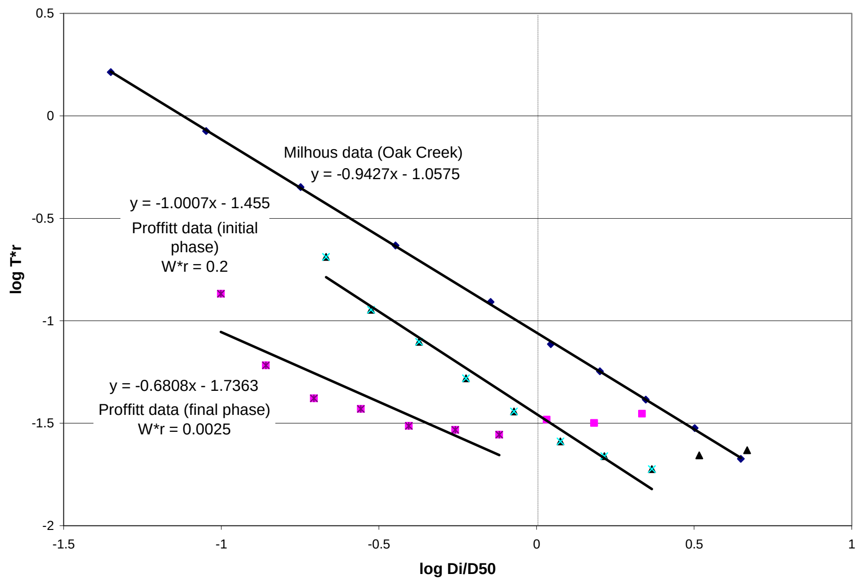

Figure 5: Variation of reference dimensionless shear stress τ_r^* with D_i/D_{50} for Oak Creek (W_r^*= 0.0025) and both initial (W_r^*= 0.2) and final phases (W_r^*= 0.0025) of Proffitt's bedload data

Figure 5 plots also the data of Oak Creek obtained by Milhous as presented and calculated by Parker, et al[8] and Diplas[6] (subsurface-based analysis). They have used W_r^* = 0.0025 and one grain size distribution with exact value for D_{50}. The Oak Creek data showed the expected linear relation on the log-log scale indicating a value β = -0.943. The initial phase of Proffitt's data has β = -0.68, which indicates non-equal mobility conditions to the favor of finer material. The value W_r^* = 0.2 has resulted β = -1 indicating the case of equal mobility for the final phase bedload data. Using W_r^* = 0.0025 for final phase data has resulted β value of -1.67 indicating non-equal mobility conditions to the favor of the coarser material.

A possible explanation of the difference between the plots of Oak Creek data and Proffitt's experimental data is that for Oak Creek, D_{50} = 20 mm, one distribution was used to represent the grain size of the bed material. For Proffitt's data, 16 different grain size distributions have been used as bed material for the final phase and 4 for the initial phase. D_{50} is the average of the different grain size distributions used in the analysis. The other possible explanation is the selection of W_r^*. For Oak Creek W_r^* = 0.0025 bounds all W_r^* values of the 10 grain size ranges. For Proffitt's data, W_r^* = 0.0025 bounds only part of the final phase data. It is, therefore, necessary to further investigate the effect of choosing W_r^* on the above relation and the similarity collapse approach. Proffitt's final phase bedload data has showed the need to select a value that is within the lower range of the measurements.

CONCLUSIONS

Fractional analysis and similarity collapse approach have been applied to both initial and final phases of the bedload data that has been collected experimentally by Proffitt[7]. Both surface and subsurface-based analysis have been considered in the paper. The paper also investigated the effect of the selection of the dimensionless reference bedload transport parameter, W_r^*, on the similarity collapse and fractional analysis.

The exposure and response of bed material to the applied shear stresses and thus to bedload transport is varying and subject to change with flow conditions. Therefore the reference bedload parameters can vary accordingly for the different bedload data sets. It has been indicated that the value for the reference parameter be within the lower range of the bedload data presenting the stream or the experiment conditions.

The paper has proved that fractional analysis is referenced to the size distribution of the surface rather than the subsurface. The similarity collapse technique has proved to be sensitive to the selection of the W_r^* parameter, which should be selected to vary as to the Shields stress values and flow conditions. It should not necessarily represent conditions slightly above the threshold of motion and can be higher.

The results of the fractional analysis and the selection of the reference bedaload parameter have been used to evaluate the conditions of equal versus non-equal mobility of the bedload in gravel streams. The results have consistency with the actual and physical conditions of the data.

REFERENCES

1) Parker, G. (1990). Surface-based bedload transport relation for gravel rivers. *Journal of Hydraulic Research* **28(4)**: 417-436.
2) Almedeij, H. J. & Diplas, P. (2003). Bedload transport in gravel-bed streams with unimodal sediment. *Journal for Hydraulic Engineering*, ASCE **129(11)**: 896-904.
3) Wilcock, P.R. & McArdell J.B. (1993). Surface-based fractional transport rates: Mobilization thresholds and partial transport of sand-gravel sediment. *Water Resources Research*, **29(4)**: 1297-1312.
4) Wilcock, P.R. and Crowe, J. C. (2003). A surface-based transport model for mixed-size sediment. *Journal of the Hydraulics Division*, ASCE **129(2)**: 120-128.
5) Parker, G., Klingeman, P.C., & McLean, D.G. (1982). Bedload and size distribution in paved gravel-bed streams. *Journal of the Hydraulics Division*, ASCE **108 (4)**: 544-571.
6) Diplas, P. (1987). Bedload transport in gravel-bed streams. *Journal of Hydraulic Engineering*, ASCE **113(3)**: 277-292.
7) Proffitt, G.T. (1980). Selective transport and armouring of non-uniform alluvial sediments. Report no. **80/22**, Department of Civil Engineering, University of Canterbury, New Zealand.
8) Bridge, J.S. and Bennet, S.J. (1992). A model for the entrainment and transport of sediment grains of mixed sizes, shapes and densities. *Water Resources Research* **28(2)**: 337-363.

9) Yang, C.T. (1996). "Sediment Transport Theory and Practice", McGraw Hill book Company, New York, USA.
10) Parker, G. & Toro-Escobar, C.M. (2002). Equal mobility of gravel in streams: The remains of the day. *Water Resources Research* **38(11)**, 1264: 46 (1-8).
11) Diplas, P. (1992). Effects of mixture properties on transport dynamics. In Billi, Hey, Thorne & Tacconi (eds), "Dynamics of gravel bed rivers", 131-135. John Willy and Sons., New York, USA.