An - Najah Univ. J. Res. (N. Sc.) Vol. 23, 2009

Investigation of the Propagation Characteristics of Magnetostatic Surface Waves in a Layered Structure of Left-Handed Materials (LHM) and ferrite (YIG)

دراسة خاصية انتشار الموجات الاستاتيكية المغناطيسية غير الخطية عبر طبقتين (LHM) والأخرى من مادة (YIG) الفريت

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Received: (18/12/2006), Accepted: (5/8/2008)

Abstract

The characteristics of magnetostatic surface waves at microwave frequencies in a layered structure of left-handed material film and a semi-infinite linear ferrite substrate have been investigated. The general dispersion relation is derived and analyzed numerically. It is found that it has two solutions for ω(k), one represents a physical solution and other unacceptable. The effects of the applied external magnetic field around the proposed region have also been examined.

Keywords: left-handed materials; dispersion relation; magnetostatic surface waves; wave-guides; boundary conditions.

ملخص

لقد تمت دراسة خاصية انتشار الموجات الاستاتيكية المغناطيسية غير الخطية عبر طبقتين إحداهما من مادة الفريت (YIG) و الأخرى من مادة (LHM) عند ترددات الموجات القصيرة (microwaves frequency) حيث تم اشتقاق معادلة التشتت لتحديد ثابت الانتشار المركب

بطريقة التحليل العددي. لقد تبين من خلال التحليل أن للمعادلة حلين $\omega(k)$ إحداها يمثل حلا فيزيائيا والآخر غير مقبول. كذلك تمت دراسة تأثير تطبيق المجال المغناطيسي الخارجي عند منطقة التردد المسموح فيها وحولها لتبيان سلوك المادة وتصرفها.

Introduction

Left-handed materials (LHMs)[1-7] have been receiving an increasing interest because of their proposed applications in future microwave engineering technology. Recently, Shelby et al[1] have demonstrated a two dimensionally isotropic left-handed material which consists of a two dimensionally periodic array of copper split ring resonators and wires (SRR). The LHM are at certain band of frequencies, behave with negative $\varepsilon(\omega)$ and $\mu(\omega)$, and also have imaginary part, thereby the refractive index of such metamaterial exhibits a negative value in this frequency as reported by Veselago[2]. A LHM also verify some of the explicit prediction such as reversed refraction, backward, Cherenkove radiation and reversed Doppler effect[3]. The study of nonlinear optical effects in the various waveguides structures containing gyromagnetic media such as ferrite[8] is also considered a key problem of the simulation of a number of opto-microwave electronic devices.

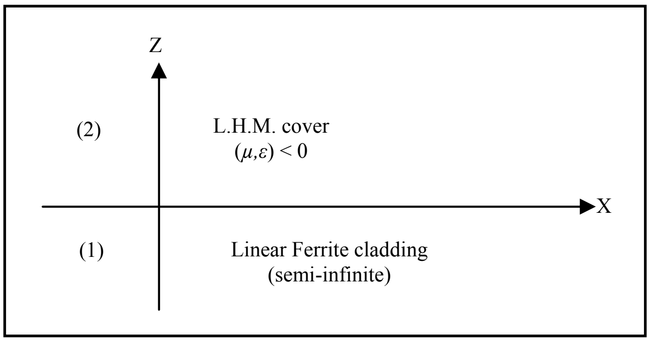


Fig. (1): Coordinate system for the single interface between LHM and a linear ferrite cladding, the applied magnetic field is in the Y-direction.

In this paper, we investigate the propagation characteristics of nonlinear magnetostatic surface waves at microwave frequencies in a layered structure of left-handed metamaterial cover and a linear ferrite substrate as shown in fig. 1.

Basic Equations

The guiding structure to be considered consists of a LHM film which is characterized by[6]:

$$\mu_{eff}(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2} \quad , \quad \varepsilon_{eff}(\omega) = 1 - \frac{\omega_p^2}{\omega^2} \tag{1}$$

Where $F = 0.56$, $\omega_0 / 2\pi = 4$ GHz , and $\omega_p / 2\pi = 10$ GHz

And, a gyromagnetic ferrite (YIG) substrate is described by a magnetic permeability tensor as (7 -8) :

$$\mu(\omega) = \begin{pmatrix} \mu_{xx} & 0 & \mu_{xz} \\ 0 & \mu_B & 0 \\ -\mu_{xz} & 0 & \mu_{xx} \end{pmatrix} \tag{2}$$

Where,

$$\mu_{xx} = \mu_B \left(\frac{\omega_0(\omega_0 + \omega_m) - \omega^2}{\omega_0^2 - \omega^2} \right), \quad \mu_{xz} = i\mu_B \frac{\omega\omega_m}{\omega_0^2 - \omega^2} \tag{3}$$

and μ_B is the usual Polder tensor elements, ω is the angular frequency of the supported wave, $\omega_0 = \gamma\mu_0 H_0$, $\omega_m = \gamma\mu_0 M_0$, H_0 is the applied magnetic field, $\gamma = 1.76\times10^{11}\ S^{-1}T^{-1}$ is the gyromagnetic ratio, M_0 is the dc saturation magnetization of the magnetic insulator and μ_B has been introduced as the background, optical magnon permeability.

The electric and the magnetic field of TE wave propagating in the x-direction can be written as:

$$E = \left(0, E_y, 0\right)\exp\left[ik_0\left(\beta x - ct\right)\right] \tag{4}$$

$$H = \left(H_x, 0, H_z\right)\exp\left[ik_0\left(\beta x - ct\right)\right] \tag{5}$$

Where $\beta = \frac{k}{k_0}$ is the complex effective wave index constant, k_0 is the wave number of the free space, and c is the velocity of light in free space.

In the ferrite substrate

The magnetostatic potential Ψ of the magnetostatic surface waves in the YIG film is written [8] as:

$$\Psi^{(1)} = A\exp\left(kz\right)e^{i(kx - \omega t)} \tag{6}$$

The relevant component of the magnetic fields for the TE magnetostatic waves in the YIG can be written after considering the phase difference as:

$$h_x^{(1)} = ikA\exp\left(kz\right)e^{i(kx - \omega t)} \tag{7}$$

$$h_z^{(1)} = -ikA\exp\left(kz\right)e^{i(kx - \omega t)} \tag{8}$$

$$e_y^{(1)} = \frac{\omega\mu_0}{k}\left(-S\mu_{xz}h_x^{(1)} + \mu_{xx}h_z^{(1)}\right) \tag{9}$$

Where, $S = \pm 1$, S = 1 stands for the propagation of the waves in forward direction, and S = -1 for the backward direction.

In LHM-cover

Maxwell's Equations are:

$$\nabla \times E = -i\omega\mu_0\mu_{eff}(\omega)H \tag{10}$$

$$\nabla \times H = i\omega\varepsilon_0\varepsilon_{eff}(\omega)E \tag{11}$$

Where the effective permeability and the effective permittivity both are less than zero.

From Eq. (10) we get:

$$\begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ ik & 0 & \frac{\partial}{\partial z} \\ 0 & E_y & 0 \end{pmatrix} = i\omega\mu_0\mu_{eff}(\omega) \begin{pmatrix} H_x \\ 0 \\ H_z \end{pmatrix} \tag{12}$$

The components of the electric field and magnetic field are:

$$\frac{-\partial E_y}{\partial z} = i\omega\mu_0\mu_{eff}H_x \tag{13}$$

Then we get:

$$H_x = \frac{i}{\omega\mu_0\mu_{eff}}\frac{\partial E_y}{\partial z} \tag{13a}$$

Similarly,

$$ikE_y = i\omega\mu_0\mu_{eff}H_z \tag{14}$$

Then we get:

$$H_z = \frac{k}{\omega\mu_0\mu_{eff}}E_y \tag{14a}$$

usingn Eq. (11), the components of magnetic field are:

$$\begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ ik & 0 & \frac{\partial}{\partial z} \\ H_x & 0 & H_z \end{pmatrix} = -i\omega\varepsilon_0\varepsilon_{eff}(\omega)\begin{pmatrix} 0 \\ E_y \\ 0 \end{pmatrix} \qquad (15)$$

Then we get:

$$-ikH_z + \frac{\partial H_x}{\partial z} = -i\,\omega\varepsilon_0\varepsilon_{eff}E_y \qquad (16)$$

Substituting both Eq's. (13a) and (14a) in Eq. (16) respectively, we obtain:

$$-ik\left(\frac{k}{\omega\mu_0\mu_{eff}}\right)E_y + \frac{\partial}{\partial z}\left[\left(\frac{i}{\omega\mu_0\mu_{eff}}\right)\frac{\partial E_y}{\partial z}\right] = -i\omega\varepsilon_0\varepsilon_{eff}E_y \qquad (17)$$

Multiplying Eq. (17) by $\omega\mu_0\mu_{eff}$, we get:

$$\frac{\partial^2 E_y}{\partial z^2} - \left(k^2 - \omega^2\mu_0\varepsilon_0\mu_{eff}\varepsilon_{eff}\right)E_y = 0 \qquad (18)$$

But $k_0^2 = \frac{\omega^2}{c^2}$ where, $\frac{1}{c^2} = \varepsilon_0\mu_0$ (18a)

And $k = k_0\beta$ (18b)

Substituting both Eq's. (18a) and (18b) in Eq. (18), we obtain:

$$\frac{\partial^2 E_y}{\partial z^2} - \left(k_0^2\beta^2 - k_0^2\mu_{eff}\varepsilon_{eff}\right)E_y = 0 \qquad (19)$$

Let $k_1^2 = k_0^2\left(\beta^2 - \mu_{eff}\varepsilon_{eff}\right)$ (19a)

Finally, we get on a 2nd order diff. eq. of the form:

$$\frac{\partial^2 E_y}{\partial z^2} - k_1^2 E_y = 0 \tag{20}$$

And one of its solution is:

$$E_y = Be^{k_1 z} \quad \text{and} \quad k_1 = k_0\sqrt{\beta^2 - \mu_{eff}\varepsilon_{eff}} \tag{20a}$$

Where, B is a constant

The relevant components of magnetic fields and the electric field in LHM have the form:

$$H_x^{(2)} = \frac{iBk_1}{\omega\mu_0\mu_{eff}} e^{k_1} e^{i(k_1 x - \omega t)} \tag{21}$$

$$H_z^{(2)} = \frac{Bk_1}{\omega\mu_0\mu_{eff}} e^{k_1 z} e^{i(k_1 x - \omega t)} \tag{22}$$

$$E_y^{(2)} = Be^{k_1 z} e^{i(k_1 x - \omega t)} \tag{23}$$

With $k_1 = k_0\sqrt{\beta^2 - \mu_{eff}\varepsilon_{eff}}$ (23a)

But for TE-waves it can be shown that[9] there is a $\frac{\pi}{2}$ phase difference between H_x and H_z. It is comfortable to redefine the field components as:

$h_x = h_x$, $h_z = ih_z$ and $e_y = ie_y$, so the field components can be written in the left handed material cover as:

$$H_x^{(2)} = \frac{iBk_1}{\omega\mu_0\mu_{eff}} e^{k_1 z} e^{i(k_1 x - \omega t)} \tag{24}$$

$$H_z^{(2)} = \frac{iBk_1}{\omega\mu_0\mu_{eff}} e^{k_1 z} e^{i(k_1 x - \omega t)} \tag{25}$$

$$E_y = iBe^{k_1 z} e^{i(k_1 x - \omega t)} \tag{26}$$

Applying the boundary conditions for the continuity of tangential H at $z = 0$ we get:

$h_x^{(1)} = H_x^{(2)}$, then we have:

$$kAe^{i(kx - \omega t)} = \frac{Bk_1}{\omega\mu_0\mu_{eff}} e^{i(k_1 x - \omega t)} \tag{27}$$

And, $e_y^{(1)} = E_y^{(2)}$, then we have:

$$-\omega\mu_0 A(S\mu_{xz} + \mu_{xx})e^{i(kx - \omega t)} = Be^{i(k_1 x - \omega t)} \tag{28}$$

Dividing Eq. (27) by Eq. (28) we obtain:

$$\frac{k}{k_1} = \frac{-1}{\mu_{eff}}(S\mu_{xz} + \mu_{xx}) \tag{29}$$

With, $k_1 = k\sqrt{\beta^2 - \varepsilon_{eff}\mu_{eff}}$ and $k = k_0\,\beta$ (29a)

Substituting Eq. (29a) in Eq. (29) and simplifying to get on the general dispersion equation.

$$\beta^2 = \frac{\varepsilon_{eff}\mu_{eff}\left(S\mu_{xz}+\mu_{xx}\right)\left(S\mu_{xz}+\mu_{xx}\right)}{\left(S\mu_{xz}+\mu_{xx}\right)\left(S\mu_{xz}+\mu_{xx}\right)-\mu_{eff}^2} \tag{30}$$

Numerical Results and discussions:

Numerical computations are performed in order to calculate the propagation characteristics of the nonlinear dispersion equation. The numerical calculations were carried out with the same parameters for the ferrite (YIG) substrate as in ref. [8]. the data parameters for the LHM as in ref.[5] ;$F = 0.56$, $\omega_0 / 2\pi = 4$ GHz , and $\omega_p / 2\pi = 10$ GHz.

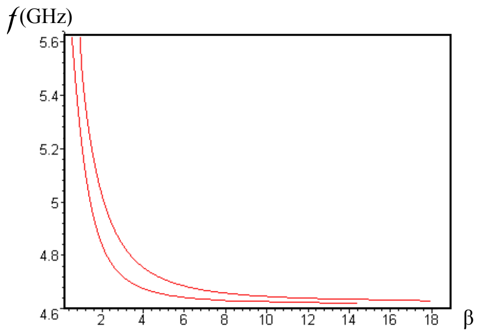
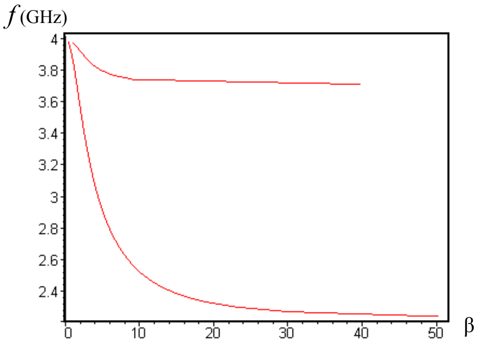


Fig. (2): Dispersion curves in both forward, backward direction, $\mu_0 H_0 = 0.5$ T, $\mu_B = 1.25$, $\mu_0 M_0 = 0.1750$ T, $\gamma = 1.76 \times 10^{11}$ rad s^{-1} T^{-1}, $\frac{\omega_p}{2\pi} = 10\ GHz$, $\frac{\omega_0}{2\pi} = 4\ GHz$, F $= 0.56$.

In this case, the frequency range where both ε(ω) and μ(ω) are negative, extends from ω_o up to 6 (GHz), where the refractive index is expected to take a negative value. Moreover, Fig.2 shows the linear dispersion curves which is in terms of the variation of the frequency with the wave index displaying the expected reciprocal behavior. This is important in microwave signal processing technology, where the propagation characteristics both forwards to the right and symmetrical. The figure shows that the frequency is decreasing with increasing the wave number k and this happen only in the propagation of TE-magnetostatic surface waves, which is in contrast with behavior of magnetostatic surface waves propagating along the interface of ferrite and normal media. It is also noticed that, by increasing the applied external magnetic field, $\mu_0 H_0$ by 0.52T, 0.55T and 0.6T respectively, the TE surface waves disappeared in the proposed region, $4 \le f \le 10$ (GHz). For $f < 4$(GHz), where $\mu_v > 0$ and $\varepsilon < 0$, the medium is transparent medium and the guiding structure behaves as metallic medium as in Fig. 3, and for $6 < f < 10$ (GHz) there is no physical solution while for $f \ge 10$ (GHz), where ($\mu_v > 0$, $\varepsilon > 0$), the physical solutions are starting to appear and the medium behaves as dielectrics as shown in fig.4.



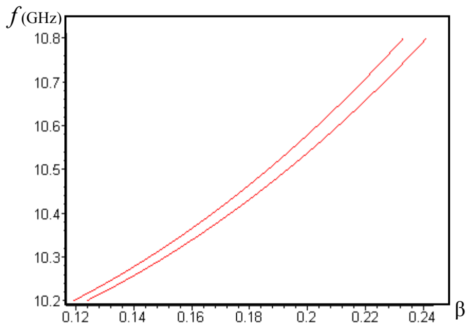


Fig. (3): Shows the dispersion characteritics considered as a metal in case ($f < 4$ GHz), $\mu_0 H_0 = 0.5$ T, $\mu_B = 1.25$, $\mu_0 M_0 = 0.1750$ T, $\gamma = 1.76 \times 10^{11}$ rad $s^{-1}T^{-1}$, $\frac{\omega_p}{2\pi} = 10\ GHz$, $\frac{\omega_0}{2\pi} = 4\ GHz$, F= 0.56.

Fig. (4): Shows the dispersion characteristics considered as a dielectric in case ($f > 6$ GHz), $\mu_0 H_0 = 0.5$ T, $\mu_B = 1.25$, $\mu_0 M_0 = 0.1750$ T, $\gamma = 1.76 \times 10^{11}$ rad $s^{-1}T^{-1}$, $\frac{\omega_p}{2\pi} = 10\ GHz$, $\frac{\omega_0}{2\pi} = 4\ GHz$, F= 0.56.

On the other hand, if we increased the applied external magnetic field $\mu_0 H_0$ by 0.2T and 0.3T for $f \le 4$(GHz) as shown in fig. 5 and for

$f \geq 10$ (GHz), as shown in fig. 6, it is seen that the propagation in the forward direction began to decrease respectively.

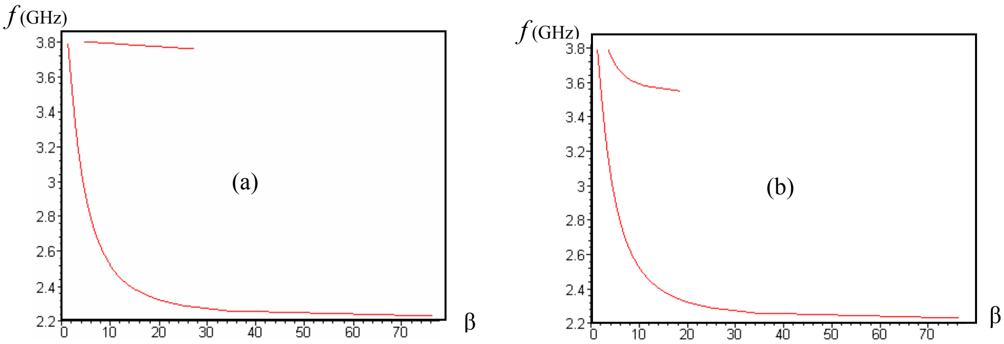


Fig.(5a-b): Shows the dispersion characteristics in case ($f < 4$ GHz), $\mu_0 H_0$ = 0.2T, 0.3T respectively, μ_B = 1.25, $\mu_0 M_0$ = 0.1750 T, γ =1.76 × 10^{11} rad $s^{-1}T^{-1}$, $\frac{\omega_p}{2\pi} = 10\ GHz$, $\frac{\omega_0}{2\pi} = 4\ GHz$, F= 0.56

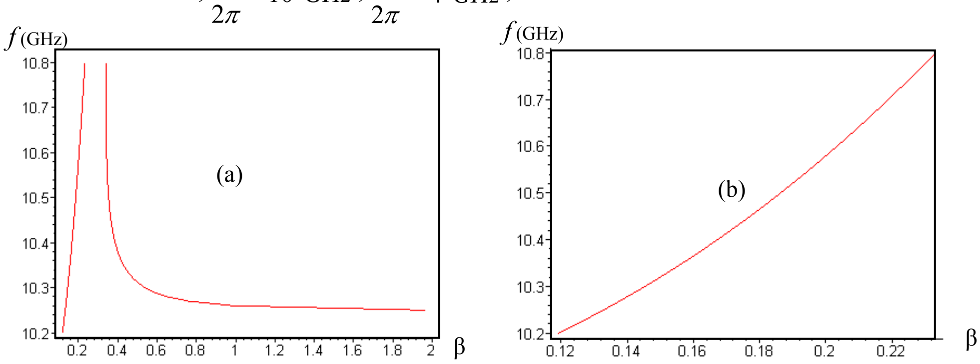


Fig.(6a-b): Shows the dispersion characteristics in case ($f \geq 10$ GHz), $\mu_0 H_0$ = 0.2T, 0.3T respectively, μ_B = 1.25, $\mu_0 M_0$ = 0.1750 T, γ =1.76 × 10^{11} rad $s^{-1}T^{-1}$, $\frac{\omega_p}{2\pi} = 10\ GHz$, $\frac{\omega_0}{2\pi} = 4\ GHz$, F= 0.56.

Conclusion

A general dispersion equation that describes the propagation of magnetostatic surface waves in a YIG film, bounded by a left handed cover, has been derived theoretically. We proposed here an approach describing a new type of reciprocal and nonreciprocal behavior in a two-layer structure, which is very promising for opto-microwave electronic devices.

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