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Ordinary and Bayesian Shrinkage Estimation

التقدير باستخدام طريقتي التقلص العادية والتقلص لبييز

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Abstract

In this paper a variety of shrinkage methods for estimating unknown population parameters has been considered. Aprior distribution for the parameters around their natural origins has been postulated and the ordinary Bayes estimators are used in place of natural origins in the ordinary shrinkage estimators to obtain Bayesian shrinkage estimators. The results are applied to the problem of estimating the location and scale parameters and the reliability function of the two-parameter exponential distribution. Simulation experiments are used to study the performances of these estimators.

Key words: Estimation, parameter, reliability, shrinkage, Bayes, guess value, exponential distribution, simulation.

ملخص

يهدف هذا البحث الى ايجاد تقديرات للمعلمتين والفاعلية للتوزيع الاسي ذي المعلمتين اعتمادا على طريقتي التقلص العادية والتقلص لبييز.

1. Introduction

In the estimation of an unknown parameter, some form of a prior knowledge about the parameter which one would like to utilize in order to get a better estimate often exists. Thompson (1968) described a shrinkage technique for estimating the mean of a population. Mehta and Srinivasan (1971) proposed another class of shrunken estimator for the mean of a population and showed that this class had a better performance than that of Thompson in terms of mean squared error. Pandey and Singh (1977) and Pandey (1979) described shrinkage techniques for estimating the variance of a normal population. Lemmer (1981) gave the concept of using an ordinary Bayes estimator instead of natural origin in the ordinary shrinkage estimator and thus derived the concept of Bayesian shrinkage estimation. He considered the estimation of binomial, Poisson and normal parameters through Bayesian shrinkage techniques. Pandey and Upadhyay(1985) considered the Bayesian shrinkage estimation of reliability of one-parameter exponential failure model.Yousef (1986) proved that the mean squared error of Thompson type estimator is smaller than the remaining shrinkage estimators for estimating the parameters of the two-parameter exponential distribution.Yousef (1991) derived confidence bounds for reliability of the two-parameter exponential distribution.

In this paper we consider the problem of estimating the parameters θ, μ and the reliability function $R(t)$ of the two-parameter exponential distribution when the prior information regarding θ, μ and $R(t)$ is available in the form of guess values. More specifically, it is assumed that the guessed values θ_0, μ_0 and $R_0(t)$ are close or approximately equal to the true values of θ, μ and $R(t)$, respectively. A variety of shrinkage methods proposed by Thompson (1968), Mehta and Srinivasan (1971), Pandey (1979) and Lemmer (1981) are used for this purpose. We propose the corresponding Bayesian shrinkage estimators of θ, μ and $R(t)$ after deriving the expressions for their ordinary Bayes estimators from type II censored sample of life testing data from the two-parameter exponential distribution.

Simulation experiments are used to study the performances of these estimators.

2. Ordinary Shrinkage Estimators

Let X be the life length of a certain system which has the probability density function $f(X;\theta,\mu) = \frac{1}{\theta}\exp[-(X-\mu)/\theta]$, $0 \le \mu \le X$, $\theta > 0$.

Then the reliability function of this system at time t is defined by $R(t) = \exp[-(t-\mu)/\theta]$.

Let us consider a random sample of n items of such a system subjected to test and the test terminated as soon as the first r ($\le$ n) items fail. Let $\underline{X} = \{X_{(1)}, X_{(2)}, \ldots, X_{(r)}\}$ be the first r ordered failure times.

It is reasonable to take the minimum variance unbiased estimators $\hat{\theta}$, $\hat{\mu}$ and $\hat{R}(t)$ of θ, μ and R(t) respectively, and modified these estimators by moving them closer to θ, μ and R(t) so that the resulting estimators, perhaps biased, have smaller mean squared error than that of $\hat{\theta}$, $\hat{\mu}$ and $\hat{R}(t)$.

It is well known from Epstein and Sobel (1954) and Basu (1964) that

$$\left.\begin{aligned}
&\hat{\theta} = \Big[\sum_{i=1}^{r} X(i) + (n-r)X_{(r)} - nX_{(1)}\Big]/(r-1), \quad r>1,\\
&\hat{\mu} = X_{(1)} - \hat{\theta}/n,\\
&\text{and}\\
&\hat{R}(t) = \frac{n-1}{n}\left[1 - \frac{t - X(1)}{(r-1)\hat{\theta}}\right]^{(r-2)}, \quad r>1,
\end{aligned}\right\} \rightarrow \quad (2.1)$$

are the minimum variance unbiased estimators of the parameters θ, μ and R(t), respectively. The variances of these estimators (see Lee (1978), p 163), are given by

$$\mathrm{var}(\hat{\theta}) = \frac{\theta^2}{r-1}, \quad r>1,$$

$$\mathrm{var}(\hat{\mu}) = \frac{r\,\theta^2}{n^2(r-1)}, \quad r>1,$$

and

$$\mathrm{var}(\hat{R}(t)) =$$

$$\frac{(n-1)^2}{n^2\Gamma(r-1)}\left[\sum_{i=0}^{2r-4}\binom{2r-4}{i}\frac{i!\,\Gamma(r-i+1)}{n^i}\sum_{m=0}^{i}\frac{\left(\frac{n(\mu-t)}{\theta}\right)^m}{m!}\right]-R^2(t), \quad r>1.$$

The first estimator considered is

$$\hat{\mu}_T = \mu_0 + c\,(\hat{\mu} - \mu_0), \quad 0 \le c \le 1, \tag{2.2}$$

where μ_0 is the guessed value of μ and $\hat{\mu}_T$ is the actual Thompson type estimator. Thompson (1968) suggested that c determined from

$\frac{\partial \mathrm{MSE}(\hat{\mu}_T)}{\partial c} = 0$ with $\mathrm{MSE}(\hat{\mu}_T) = E(\hat{\mu}_T - \mu)^2$, the mean squared error of $\hat{\mu}_T$.

It follows that

$$c = (\mu - \mu_0)^2 / [\,(\mu - \mu_0)^2 + \mathrm{var}(\hat{\mu})\,].$$

In practice c is estimated by replacing the unknown parameters by their sample estimates. Substituting the estimated value of c in (2.2) we have

$$\hat{\mu}_T = \mu_0 + (\hat{\mu} - \mu_0)^3 / [(\hat{\mu} - \mu_0)^2 + r\,\hat{\theta}^2 /(n^2 (r-1))].$$

For any value of c, $0 \leq c \leq 1$, and when μ_0 tends to μ, it is easily seen from (2.2) that

$$\text{MSE}(\hat{\mu}_T) = c^2 \text{var}(\hat{\mu}) + (1-c)^2(\mu - \mu_0)^2 \leq \text{var}(\hat{\mu}).$$

Secondly, we consider the Mehta and Srinivasan (1971)-type estimator. This is given by

$$\hat{\mu}_M = \hat{\mu} - a(\hat{\mu} - \mu_0)\exp[-b(\hat{\mu} - \mu_0) / \text{var}(\hat{\mu})], \tag{2.3}$$

where a and b are positive constants to be suitably chosen such that $0<a<1$ and $b>0$. No general guidance has been given on how a and b should be chosen. Substituting var $(\hat{\mu})$ and unknown parameters by their sample estimates in (2.3) we obtain

$$\hat{\mu}_M = \hat{\mu} - a(\hat{\mu} - \mu_0) \exp[-bn^2(r-1)(\hat{\mu} - \mu_0)/r\,\hat{\theta}^2].$$

It can be verified from (2.3) that the minimum and maximum values of $\hat{\mu}_M$ is attainable when b tends to 0 and ∞ respectively by a suitable choice of a, $0<a<1$. So we take

$$\lim_{b \to 0} \text{MSE}(\hat{\mu}_M) = (1-a)^2 \text{ var}(\hat{\mu}) + a(\mu - \mu_0)^2,$$

and

$$\lim_{b \to \infty} MSE(\hat{\mu}_M) = var(\hat{\mu}).$$

Hence for 0<a<1, b>0 and μ_0 tends to μ we have

$$MSE(\hat{\mu}_M) \leq MSE(\hat{\mu}).$$

Thirdly, we consider the Pandey (1979) - type estimator of μ is given by

$$\hat{\mu}_p = a[k\hat{\mu} + (1-k)\mu_0], \quad 0 \leq k \leq 1, \qquad (2.4)$$

with k is a constant specified by the experimenter according to his belief in μ_0 and a is determined from $\frac{\partial MSE(\hat{\mu}_p)}{\partial a} = 0$. It follows that

$a = d_1\mu^2 / [k^2 \, var(\hat{\mu}) + d_1 \mu^2]$ where $d_1 = k + (1-k)\mu_0/\mu$. Usually a is estimated by replacing the unknown parameters by their sample estimates. Substituting the estimated value of a in (2.4) we obtain

$$\hat{\mu}_P = \hat{d}_1^2 \hat{\mu}^3 / [\hat{d}_1^2 \hat{\mu}^2 + k^2 r \hat{\theta}^2 / (n^2(r-1))],$$

with $\hat{d}_1 = [k + (1-k)\mu_0/\hat{\mu}]$.

It can be shown from (2.4) that

$$MSE(\hat{\mu}_P) = ak^2 \, var(\hat{\mu}) + [(1-ak)\mu - a(1-k)\mu_0]^2.$$

It follows that $MSE(\hat{\mu}_P) \leq MSE(\hat{\mu})$ only when a=1 and μ_0 tends to μ, it is not clear otherwise.

Finally, we consider the Lemmer (1981)-type estimator for μ .

This is given by

$$\hat{\mu}_L = k\hat{\mu} + (1-k)\,\mu_0 . \qquad (2.5)$$

It can be seen from (2.4) and (2.5) that $\hat{\mu}_P = \hat{\mu}_L$ if a=1 .

All the above approaches can be used to define a variety of shrunken estimators for the parameter θ and the reliability function R (t). The estimators considered for the parameters μ, θ and R(t) are presented in Table 1.

Table (1): Shrunken estimators for μ, θ and R (t)

Parameter	Type of Estimator	Estimator
Location Parameter μ	Thompson	$\hat{\mu}_T = \mu_0 + \dfrac{(\hat{\mu}-\mu_0)^3}{(\hat{\mu}-\mu_0)^2 + r\hat{\theta}^2/n^2(r-1)}$
	Mehta-Srinivasan	$\hat{\mu}_M = \hat{\mu} - a\,(\hat{\mu}-\mu_0)\exp[-bn^2(r-1)(\hat{\mu}-\mu_0)/r\hat{\theta}^2]$
	Pandey	$\hat{\mu}_P = \dfrac{\hat{d}_1^2\,\hat{\mu}^3}{\hat{d}_1^2\,\hat{\mu}^2 + k^2 r\hat{\theta}^2/n^2(r-1)}$
	Lemmer	$\hat{\mu}_L = k\hat{\mu} + (1-k)\mu_0$

… Continue table (1)

Parameter	**Type of Estimator**	**Estimator**
Scale Parameter θ	Thompson	$\hat{\theta}_T = \hat{\theta}_0 + \dfrac{(\hat{\theta} - \theta_0)^3}{(\hat{\theta} - \theta_0)^2 + \hat{\theta}^2/(r-1)}$
	Mehta-Srinivasan	$\hat{\theta}_M = \hat{\theta} - a(\hat{\theta} - \theta_0)\exp[-b(r-1)(\hat{\theta} - \theta_0)/\hat{\theta}^2]$
	Pandey	$\hat{\theta}_P = \dfrac{\hat{d}_2^2\,\hat{\theta}^3}{\hat{d}_2^2\,\hat{\theta}^2 + k^2\,\hat{\theta}^2/(r-1)}$
	Lemmer	$\hat{\theta}_L = k\hat{\theta} + (1-k)\theta_0$
Reliability Function R(t)	Thompson	$\hat{R}_T(t) = R_0(t) + \dfrac{(\hat{R}(t) - R_0(t))^3}{(\hat{R}(t) - R_0(t))^2 + \hat{var}(\hat{R}(t))}$
	Mehta-Srinivasan	$\hat{R}_M(t) = \hat{R}(t) - a(\hat{R}(t) - R_0(t))\exp[-b(\hat{R}(t) - R_0(t))/\hat{var}(\hat{R}(t))]$
	Pandey	$\hat{R}_P(t) = \dfrac{\hat{d}_3^2\,\hat{R}(t)^3}{\hat{d}_3^2\,\hat{R}(t)^2 + k^2\,\hat{var}(\hat{R}(t))}$
	Lemmer	$\hat{R}_L(t) = k\hat{R}(t) + (1-k)R_0(t)$

where $\hat{d}_1 = k + (1-k)\ \mu_0 / \hat{\mu}$, $\hat{d}_2 = k + (1-k)\ \theta_0 / \hat{\theta}$, $\hat{d}_3 = k + (1-k) R_0(t) / \hat{R}(t)$, k is a known constant between zero and one, a and b are positive constants to be suitably chosen such that 0<a<1 and b>0, μ_0, θ_0 and $R_0(t)$ are the guessed values for μ, θ and R(t) respectively and $\hat{var}(\hat{R}(t))$ is the estimated variance of the estimator $\hat{R}(t)$ which is given by

$$var(\hat{R}(t)) = \frac{(n-1)^2}{n^2\Gamma(r-1)} \left[\sum_{i=0}^{2r-4} \binom{2r-4}{i} \frac{i!\Gamma(r-i+1)}{n^i} \sum_{m=0}^{i} \frac{\left(\frac{n(\hat{\mu}-t)}{\hat{\theta}}\right)^m}{m!} \right] - \hat{R}^2(t), \quad r>1.$$

3. Bayesian Shrinkage Estimators

Using the set up of section 2, the likelihood function of $\underline{X}$ is given by

$$L(\underline{X} / \theta, \mu) = \frac{n!}{(n-r)!\theta^r} \exp[-(r\hat{\theta} + n\hat{\mu} - n\mu)/\theta] .$$

Assume that our prior knowledge about θ and μ can be expressed as

$$g(\theta,\mu) = \frac{\beta^\alpha}{\delta\,\Gamma\alpha} \left(\frac{1}{\theta}\right)^{\alpha+1} \exp[-\beta/\theta], \quad \alpha, \beta, \delta, \theta > 0, \quad 0 \le \mu \le \delta,$$

where α, β, δ are known constants. Combining the above prior with the likelihood function we obtain the posterior probability density function of θ and μ as

$$h(\theta, \mu / \underline{X}) = S^{-1} \left(\frac{1}{\theta}\right)^{r+\alpha+1} \exp[-(r\hat{\theta} + n\hat{\mu} - n\mu + \beta)/\theta], \ \alpha, \beta, \delta, \theta>0, \ o \le \mu \le M,$$

where

$M = \min(\delta, X_{(1)})$,

and

$$S = \frac{\Gamma(r+\alpha-1)}{n}\left[\frac{1}{R_1^{r+\alpha-1}} - \frac{1}{R_2^{r+\alpha-1}}\right],$$

with $R_1 = \sum_{i=1}^{r} X(i) + (n-r)X_{(r)} - nM + \beta$ and $R_2 = R_1 + nM$.

Bayes estimators of θ, μ and R(t) with respect to the squared error loss function are given by

$$\hat{\theta}_0 = \int\int \theta\, h(\theta, \mu/\underline{X})\, \partial u\, \partial\theta = \frac{\Gamma(r+\alpha-2)}{n\,s}\left[\frac{1}{R_1^{r+\alpha-2}} - \frac{1}{R_2^{r+\alpha-2}}\right],$$

$$\hat{\mu}_0 = \int\int \mu\, h(\theta, \mu/\underline{X})\, \partial\theta\, \partial u = \frac{\Gamma(r+\alpha-2)}{n^2 s}\left[\frac{(r+\alpha-1)nM - R_2}{R_1^{r+\alpha-1}} - \frac{1}{R_2^{r+\alpha-1}}\right],$$

and

$$\hat{R}_0(t) = \int\int e^{-((t-\mu)/\theta)}\, h(\theta, \mu/\underline{X})\, \partial u\, \partial\theta$$

$$= \frac{\Gamma(r+\alpha-1)}{(n+1)s}\left[\frac{1}{(R_1 + t - M)^{r+\alpha-1}} - \frac{1}{(R_2 + t)^{r+\alpha-1}}\right], \qquad \textbf{(3.1)}$$

respectively.

If we substitute the Bayes estimators $\hat{\theta}_0$, $\hat{\mu}_0$ and $\hat{R}_0(t)$ in place of natural origins θ_0, μ_0 and $R_0(t)$ in a shrunken estimators presented in Table1, we obtain the Bayesian shrunken estimators for the parameters θ, μ and R(t) . For example

$$\hat{\mu}_T^B = \hat{\mu}_0 + (\hat{\mu} - \hat{\mu}_0)^3 / [(\hat{\mu} - \hat{\mu}_0)^2 + r\,\hat{\theta}^2 / n^2 (r-1)],$$

$$\hat{\mu}_M^B = \hat{\mu} - a(\hat{\mu} - \hat{\mu}_0)\exp[-bn^2(r-1)(\hat{\mu} - \hat{\mu}_0)/r\hat{\theta}^2],$$

$$\hat{\mu}_P^B = d_1\hat{\mu}^2\hat{\mu}^3 / [d_1\hat{\mu}^2\hat{\mu}^2 + k^2 r\hat{\theta}^2/n^2(r-1)] \text{ with } \hat{d}_1 = k + (1-k)\hat{\mu}_0/\hat{\mu},$$

$$\hat{\mu}_L^B = k\hat{\mu} + (1-k)\hat{\mu}_0.$$

are Thompson, Metha-Srinivasan, Pandey and Lemmer Bayesian shrinkage estimators respectively for the parameter μ. In the same manner we can find the other types of Bayesian shrinkage estimators for the parameters θ and reliability function R(t).

4. Simulation

The researcher uses simulation experiments to study the performances of the estimators obtained in Sections 2 and 3.A random sample of size n from the two-parameter exponential distribution with μ =80 and θ=7 is generated . The vector $\underline{X} = \{X_{(1)}, X_{(2)}, \ldots, X_{(r)}\}$ of the first r-ordered observation is recorded. Then the minimum variance unbiased estimators $\hat{\mu}$, $\hat{\theta}$ and $\hat{R}(t)$ of μ, θ and R(t) respectively are computed using the formulas in (2.1). For a known constant k between zero and one and for specific values μ_0, θ_0 and $R_0(t)$ the quantities

$\hat{d}_1 = k+(1-k)\ \mu_0/\hat{\mu}$, $\hat{d}_2 = k+(1-k)\ \theta_0/\hat{\theta}$, $\hat{d}_3 = k+(1-k)\ R_0(t)/\hat{R}(t)$ are obtained. Then the ordinary shrinkage estimators $\hat{\mu}_T$, $\hat{\mu}_M$, $\hat{\mu}_p$ and $\hat{\mu}_L$ of μ, $\hat{\theta}_T$, $\hat{\theta}_M$, $\hat{\theta}_p$ and $\hat{\theta}_L$ of θ, and $\hat{R}_T(t)$, $\hat{R}_M(t)$, $\hat{R}_P(t)$ and $\hat{R}_L(t)$ of R(t) are computed using the corresponding formulas shown in Table 1. For given values of α, β, and δ, the Bayesian estimators $\hat{\mu}_0$, $\hat{\theta}_0$ and

$\hat{R}_0(t)$ of μ, θ and R(t) respectively are obtained by using the formulas given in (3.1). Then the Bayesian estimates $\hat{\mu}_0$, $\hat{\theta}_0$ and $\hat{R}_0(t)$ are obtained and substituted in place of natural origins μ_0, θ_0 and $R_0(t)$ in shrunken estimators obtained in Table 1. Thus the Bayesian shrunken estimators $\hat{\mu}_T^B$, $\hat{\mu}_M^B$, $\hat{\mu}_P^B$, and $\hat{\mu}_L^B$ of μ, $\hat{\theta}_T^B$, $\hat{\theta}_M^B$, $\hat{\theta}_P^B$ and $\hat{\theta}_L^B$ of θ and $\hat{R}_T^B(t)$, $\hat{R}_M^B(t)$, $\hat{R}_P^B(t)$, and $\hat{R}_L^B(t)$ of R(t) are computed. Monote Carlo experiments are repeated 500 times. The average of the 500 sample values of each squared error, e.g. $(\hat{\mu} - \mu)^2$, is taken as an estimate of the corresponding mean squared error which is denoted by MSE.The estimates of the mean squared errors of the various estimators of μ, θ and R(t) and the relative efficiencies, e.g.

$$R(\hat{\mu}_T / \hat{\mu}) = MSE(\hat{\mu}_T) / MSE(\hat{\mu}),$$

$$R(\hat{\mu}_T / \hat{\mu}_T^B) = MSE(\hat{\mu}_T) / MSE(\hat{\mu}_T^B),$$

are calculated for n=30, r=10,20,30, k=0.05,0.5, a=0.1,0.5, b =40, 500, $\alpha = \beta = 2$,

$\delta = 82$, $\mu = \mu_0 = 80$ and $\theta = \theta_0 = 7$. Results of the simulation experiments are given in Tables 2-7.

5. Conclusions

Although the results derived above apply strictly to limited cases, they are suggestive of some general conclusions regarding the relative efficiencies of the various methods. It can be seen from Tables 2-4 that MSE of Thompson, Mehta and Srinivasan, and Lemmer estimators are smaller than that of $\hat{\mu}$, $\hat{\theta}$ and $\hat{R}(t)$. The advantages of $\hat{\mu}_T$ and $\hat{\mu}_L$ are most marked when r is small.

Further comparison statistics in Tables 5-7 show that when the natural origins are close to the true values of μ, θ and R(t), the MSE of Thompson, Mehta and Srinivasan and Lemmer ordinary shrinkage estimators are smaller than the MSE of their corresponding Bayesian shrinkage estimators, while the MSE of Pandey Bayesian shrinkage estimators is smaller than that of the corresponding ordinary shrinkage estimator. If the natural origins are far away from the true values, then the MSE of the various ordinary shrinkage estimators is higher than that of $\hat{\mu}$, $\hat{\theta}$ and $\hat{R}(t)$, while the Bayesian shrinkage estimators still have smaller MSE than that of $\hat{\mu}$, $\hat{\theta}$ and $\hat{R}(t)$.

Table 2: Relative efficiencies of various ordinary shrunken estimators of μ with respect to $\hat{\mu}$.

Sample size n=30, $\mu = \mu_0 = 80$, $\theta_0 = 7$.

No. of failures	$R(\hat{\mu}_T / \hat{\mu})$	a=.1, b=40	a=.5, b=500	k=.05	k=.50	k=.05	k=.50
		$R(\hat{\mu}_M / \hat{\mu})$		$R(\hat{\mu}_P / \hat{\mu})$		$R(\hat{\mu}_L / \hat{\mu})$	
10	$4.43\text{x } 10^{-6}$	$6.54 \text{ x}10^{-3}$	0.544	4.707	1.258	$2.47\text{x}10^{-3}$	249
20	$2.52 \text{ x}10^{-4}$	0.226	0.998	3.788	3.276	$2.50\text{x}10^{-3}$	255
30	$1.18\text{x } 10^{-3}$	0.166	0.994	4.004	2.736	$2.49\text{x}10^{-2}$	259

Table 3: Relative efficiencies of various ordinary shrunken estimators of θ with respect to $\hat{\theta}$.

Sample size n=30, $\mu = 80$, $\theta = \theta_0 = 7$

No. of failures	$R(\hat{R}_T / \hat{\theta})$	a=.1, b=40	a=.5, b=500	k=.05	k=.50	k=.05	k=.50
		$R(\hat{\theta}_M / \hat{\theta})$		$R(\hat{\theta}_P / \hat{\theta})$		$R(\hat{\theta}_L / \hat{\theta})$	
10	$9.34\text{x } 10^{-5}$	$6.60\text{x}10^{-2}$	0.969	1.15	1.670	$2.33\text{x}10^{-4}$	.095
20	$1.48\text{ x}10^{-3}$	0.945	0.721	3.965	1.435	$2.53\text{x}10^{-3}$	.223
30	$4.13\text{x } 10^{-3}$	0.876	1.0	3.878	1.753	$2.50\text{x}10^{-2}$	.256

Table (4): Relative efficiencies of various ordinary shrunken estimators of R(t) with respect to $\hat{R}(t)$

Sample size n=30, $\mu = \mu_0 = 80$, $\theta = \theta_0 = 7$, t=85, $R(t)=R_0(t) = .490$

No. of failures	$R(\hat{R}_T(t) / \hat{R}(t))$	a=.1, b=40	a=.5, b=500	k=.05	k=.50	k=.05	k=.50
		$R(\hat{R}_M(t) / \hat{R}(t))$		$R(\hat{R}_P(t) / \hat{R}(t))$		$R(\hat{R}_L(t) / \hat{R}(t))$	
10	$4.19\text{x } 10^{-5}$	$4.40\text{x}10^{-3}$	0.463	2.153	1.161	$7.81\text{x}10^{-4}$	.201
20	$2.84\text{ x}10^{-4}$	0.890	0.918	2.908	1.624	$6.17\text{x}10^{-3}$	.248
30	$3.26\text{x } 10^{-4}$	0.154	0.965	2.841	1.563	$8.30\text{x}10^{-3}$	.267

Table 5: Relative efficiencies of various ordinary shrunken estimators of μ with respect to their corresponding Bayesian shrunken estimators.

Sample size n=30, $\mu = 80$, $\theta = 7$, $\alpha = \beta = 2$, $\delta = 82$

No. of failures	$R(\hat{\mu}_T / \hat{\mu}_T^B)$	a=.1, b=40	a=.5, b=500	k=.05	k=.50	k=.05	k=.50
		$R(\hat{\mu}_M / \hat{\mu}_M^B)$		$R(\hat{\mu}_P / \hat{\mu}_P^B)$		$R(\hat{\mu}_L / \hat{\mu}_L^B)$	
10	5.66×10^{-2}	9.59×10^{-5}	8.39×10^{-3}	3.706	3.741	3.14×10^{-2}	.287
20	2.39×10^{-3}	9.32×10^{-2}	3.66×10^{-5}	3.989	2.097	2.41×10^{-2}	.249
30	1.27×10^{-2}	3.10×10^{-2}	1.56×10^{-5}	4.002	2.731	2.64×10^{-2}	.255

Table (6): Relative efficiencies of various ordinary shrunken estimators of θ with respect to their corresponding Bayesian shrunken estimators

Sample size n=30, $\mu = 80$, $\theta = 7$, $\alpha = \beta = 2$, $\delta = 82$

No. of failures	$R(\hat{\theta}_T / \hat{\theta}_T^B)$	a=.1, b=40	a=.5, b=500	k=.05	k=.50	k=.05	k=.50
		$R(\hat{\theta}_M / \hat{\theta}_M^B)$		$R(\hat{\theta}_P / \hat{\theta}_P^B)$		$R(\hat{\theta}_L / \hat{\theta}_L^B)$	
10	5.08×10^{-2}	0.918	0.992	1.428	1.823	5.96×10^{-2}	0.504
20	0.280	0.833	0.971	4.376	3.865	0.187	0.609
30	0.645	0.902	0.997	4.421	3.626	0.231	0.762

Table 7: Relative efficiencies of various ordinary shrunken estimators of R(t) with respect to their corresponding Bayesian shrunken estimators

Sample size n=30, $\mu = 80$, $\theta = 7$, t=85, R(t) = .490, $\alpha=\beta=2$, $\delta=82$

No. of failures	$R(\hat{R}_T(t) / \hat{R}_T^B(t))$	a=.1, b=40	a=.5, b=500	k=.05	k=.50	k=.05	k=.50
		$R(\hat{R}_M(t) / \hat{R}_M^B(t))$		$R(\hat{R}_P(t) / \hat{R}_P^B(t))$		$R(\hat{R}_L(t) / \hat{R}_L^B(t))$	
10	9.32×10^{-3}	2.73×10^{-3}	6.89×10^{-4}	3.673	2.896	2.26×10^{-3}	.232
20	8.81×10^{-2}	1.01×10^{-3}	7.45×10^{-3}	3.924	1.809	2.59×10^{-2}	.166
30	0.186	5.64×10^{-2}	3.45×10^{-2}	3.566	3.052	3.67×10^{-3}	.218

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