An-Najah Univ. J. Res., Vol. 15, 2001

Review of The Multiplier Algebra of Orlicz Spaces

مراجعة لجبر مضاعفات فضاء اورلكس

Mahmoud Masri

Mathematics Department, Faculty of Science, An-Najah National University, Nablus, Palestine.

Received: (9/10/2000), Accepted: (7/5/2001)

Abstract

In this review of the paper of A. Hakawati ,"The Multiplier Algebra of Orlicz Spaces" we point out a gap in the proof of the main result and suggest a proof of it ., We also point out some misprints.

ملخص

في هذة المراجعه لورقة البحث للدكتور ع. حكواتي "جبر مضاعفات اورلكس" نبين فجوة في برهان النتيجة الرئيسية ونقترح برهانا لها.كذلك سنبين بعض الأخطاء المطبعية.

Printing errors

Page number	Line number	Error	Correction
1	11↓	$\overline{\lim_{x\to 0}}\dfrac{\phi(x^2)}{\phi(x)}$	$\overline{\lim_{x\to \infty}}\dfrac{\phi(x^2)}{\phi(x)}$
1	15↓	$\overline{\lim_{x\to 0}}\dfrac{\phi(x^2)}{\phi(x)}$	$\overline{\lim_{x\to \infty}}\dfrac{\phi(x^2)}{\phi(x)}$
2	10↑	Deep	Deeb
3	2↓	function	functions
3	9↓	when ever	whenever
4	5↓	$<$	$<\infty$
4	7↓	$\overline{\lim_{x\to \infty}}\dfrac{\phi(x^2)}{\phi(x)}<\infty$	$\overline{\lim_{x\to \infty}}\dfrac{\phi(x^2)}{\phi(x)}=\infty$

Page number	Line number	Error	Correction
4	7↑	$\frac{\phi(x_n^2)}{\phi(x_n)} > m$	$\frac{\phi(x^2)}{\phi(x\ )} > m$
5	1↓	X	x
5	1↓	a	a_n
5	1↑	=	≤

The gap in the proof of the main result is in line 4 from the bottom of page 5 which is

$$\sum n\phi(y_n)l(J_n),(y_n\ is\ as\ above) = \sum \frac{1}{n}$$

where equality need not hold since $a_n \le y_n \le b_n$ and

$$l(J_m)\phi(b_m) = \frac{1}{m^2}$$

for all m is not applicable.

Suggested proof of the main result :

Suppose that $\overline{\lim_{x\to\infty}} \frac{\phi(x^2)}{\phi(x)} = \infty$ .Then there exists an increasing sequence $\{x_n\}$ such that $x_1 > 1$ and $\frac{\phi(x_n^2)}{\phi(x_n)} > n$ for all n=1,2,3,… .To see this let $g(x) = \sup\{\frac{\phi(t^2)}{\phi(t)} : t > x\}$ .Then $\lim_{x\to\infty} g(x) = \infty$ and for all n=1,2,3,…there exists y_n such that g(x)>n for all x≥ y_n .Chooze $z_1 > \max\{y_1,1\}$ and $x_1 > z_1$ such that

$1 < \dfrac{\phi(x_1^2)}{\phi(x_1)} \le g(z_1)$. For n=2,3,… choose $z_n > \max\{y_n, x_{n-1}\}$ and $x_n > z_n$ such that $n < \dfrac{\phi(x_n^2)}{\phi(x_n)} \le g(z_n)$.

Let $c = \sum_{n=1}^{\infty} \dfrac{1}{n^2\phi(x_n)}$, $J_1 = (0, \dfrac{1}{c\phi(x_1)})$ and $J_n = (\sum_{i=1}^{n-1} \dfrac{1}{ci^2\phi(x_i)}, \sum_{i=1}^{n} \dfrac{1}{ci^2\phi(x_i)})$ for all n=2,3,… .Then the length $l(J_n)$ of J_n is given by

$$l(J_n) = \frac{1}{cn^2\phi(x_n)} \text{ for all n=1,2,3,…}$$

If X=[0,1] equipped with the Lebesgue measure μ and the function f is defined on X by f(x)= x_n for x in J_n and zero otherwise then

$$\| f \|_\phi = \int_X \phi(| f |)d\mu = \sum_{n=1}^{\infty} \int_{J_n} \phi(| f |)d\mu = \sum_{n=1}^{\infty} l(J_n)\phi(x_n) = \sum_{n=1}^{\infty} \frac{1}{cn^2} < \infty$$

Moreover,

$$\| f^2 \|_\phi = \int_X \phi(| f^2 |)d\mu = \sum_{n=1}^{\infty} \int_{J_n} \phi(| f^2 |)d\mu$$

$$= \sum_{n=1}^{\infty} l(J_n)\phi(x_n^2) \ge \sum_{n=1}^{\infty} nl(J_n)\phi(x_n) = \sum_{n=1}^{\infty} \frac{1}{cn} = \infty$$

Thus $f \in L_\phi$ but $f^2 \notin L_\phi$.

Reference

A. Hakawati, The Multiplier Algebra of Orlicz Spaces, *An-Najah Univ. J. Res.*,Vol.**12**,(1998),1-6