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# Some Remarks on Closure and Strong Continuity*

# بعض الخصائص للاقترانات قوية ومغلقة الاتصال

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**Abstract**

Noire studied several properties of weak continuity in Proc. Amer. Math. Soc. 46(1), 120-124. In this paper it is shown that similar to most of the results of the above paper still hold for closure and strong continuity. Example 2 is a counterexample to a corollary to Theorem 6 of Long and Herrington. Theorem 12 of our paper is a sharper result to Theorem 5 of Noire. Several decomposition theorems of closure and strong continuity are obtained.

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**Key words:** δ-closed, θ- closed, θ- open, H-closed, closure compact, weak continuity, closure continuity, strong continuity.

يقوم المؤلف في هذا البحث بإعطاء عدة خصائص لنوعين من التعميمات للإقترانات المتصلة مشابهة للخصائص التي حصل عليها نويري للاقترانات ضعيفة الاتصال. كما يقوم المؤلف بإعطاء مثال مضاد يبين فيه عدم صحة النتيجة التي تتبع نظرية ٦ من بحث لونغ وهيرينغتون. إن نظرية ١٢ من هذا البحث هي تعميم لنظرية ٥ من بحث نويري.

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* Dedicated to Mary Anne Swardson who taught me how to enjoy mathematics.

## 1. Introduction

The concepts of $\theta$-*closure*, $\delta$-*closure*, $\theta$-*interior and* $\delta$-*interior* operators were first introduced by Velickho. These operators have since then been studied intensively by many authors. Although $\theta$-*interior and* $\theta$-*closure operators are not idempotents*, the collection of all $\delta$-open sets in a topological space $(X,\Gamma)$ forms a topology $\Gamma_\theta$ on $X$, called the *semiregularization topology of* $\Gamma$, weaker than $\Gamma$ and the class of all regular open sets in $\Gamma$ forms an open basis for $\Gamma_s$. Similarly, the collection of all $\theta$-open sets in a topological space $(X,\Gamma)$ forms a topology $\Gamma_\theta$ on X, weaker than $\Gamma_s$. So far, numerous applications of such operators have been found in studying different types of continuous like maps, separation of axioms, and above all, to many important types of compact like properties. In 1961, [6] introduced the concept of weak continuity as a generalization of continuity, later in 1966, Husain introduced almost continuity as another generalization, and Andrew and Whittlesy [2], the concept of closure continuity which is stronger than weak continuity. In 1968, Singal and Singal introduced a new almost continuity which is different from that of Husain. A few years later, P. E. Long and Carnahan [8] studied similarities and dissimilarities between the two concepts of almost continuity. The purpose of this paper is to further the study of the concepts of closure and strong continuity. We get similar results to those in [8], [11] applied to closure and strong continuity. Among other results we prove that the graph mapping of $f$ is closure continuous iff $f$ is closure continuous. In Theorem 3, we show that if the graph mapping of $f$ is strongly continuous then $f$ is strongly continuous but not conversely. Theorem 12 is a stronger result of Theorem 5 in [11]. Theorem 8 shows that a strong retraction of a Hausdorff space is $\theta$-closed. Several decomposition theorems of closure and strong continuity are given in this paper. Example 2 shows that [9, Corollary to Theorem 6] is not true.

For a set $A$ in a space $X$, let us denote by $Int(A)$ and $cls(A)$ for the interior and the closure of $A$ in $X$, respectively. Following Velickho, a point $x$ of a space $X$ is called a $\theta$-*adherent point of a subset A of X iff* $cls(U) \cap A \neq \phi$, *for every open set U containing x.* The set of all $\theta$ -adherent points of $A$ is called *the* $\theta$-*closure of A, denoted by* $cls_\theta\ A$. A subset $A$ of a space $X$ is *called* $\theta$-*closed iff* $A = cls_\theta\ A$. The complement of a $\theta$ -closed set is called $\theta$-open. Similarly, *the* $\theta$*-interior of a set A in X, written* $Int_\theta A$, *consists of those points x of A such that for some open set U containing x,* $cls(U) \subseteq A$. A set $A$ is $\theta$-*open iff* $A = Int_\theta\ A$, or equivalently, $X$-$A$ is $\theta$-closed. Clearly every $\theta$-closed ($\theta$-open) is closed (open). It is well-known that one of the most weaker forms of compactness is closure compactness (QHC). A closure compact Hausdorff space is called H-closed, first defined by Alexandroff and Urysohn.

A function f: $X \rightarrow Y$ is weakly continuous at $x \in X$ if given any open set V in Y containing f(x), there exists an open set U in X containing x such that $f(U) \subseteq cls(V)$. If this condition is satisfied at each $x \in X$, then f is said to be weakly continuous. A function $f: X \rightarrow Y$ is closure continuous ($\theta$ -continuous) at $x \in X$ if given any open set $V \subseteq Y$ containing f(x), there exists an open set U in X containing x such that $f(cls(U)) \subseteq cls(V)$. If this condition is satisfied at each $x \in X$, then f is said to be closure continuous ($\theta$-continuous). A function $f: X \rightarrow Y$ is strongly continuous (strongly $\theta$-continuous) at $x \in X$ if given any open set $V \subseteq Y$ containing f(x), there exists an open set $U \subseteq X$ containing x such that $f(cls(U)) \subseteq V$. If this condition is satisfied at each $x \in X$, then f is said to be strongly continuous (strongly $\theta$-continuous). A function $f: X \rightarrow Y$ is said to be almost continuous in the since of Singal and Singal (briefly a.c.S) if for each point $x \in X$ and each open set $V \subseteq Y$ containing f(x), there exists an open set $U \subseteq X$ containing x such that $f(U) \subseteq Int(cls(V))$. A function $f: X \rightarrow Y$ is said to be almost continuous in the since of Husain (briefly a.c.H) if for each $x \in X$ and each open set $V \subseteq Y$ containing f(x), $cls(f^{-1}(V))$ is a neighborhood of $x \in X$. A space X is called completely Hausdorff or Urysohn if for every $x \neq y \in X$, there exist an open set U containing x and an open set V containing y such that cls(U)$\cap$ cls(V)=$\phi$ .

## 2. The Results

Clearly cls(A) $\subseteq$ $cls_\theta$ A, but not equal as it is shown in the next example. Over a regular space, it is clear that cls(A)=$cls_\theta$ A.

*Example 1.* Let R be the reals with the cofinite topology. Then every finite subset of R is closed, but the θ-closure of every none empty set is R.

*Theorem 1.* Let f:X→ Y.Then the following are equivalent:

a) f($cls_\theta$ A)$\subseteq$clsf(A), for every A$\subseteq$X;
b) The inverse image of every closed is θ-closed;
c) The inverse image of every open is θ-open;
d) f is strongly continuous.

*Proof.* (a)⇒ (b). Let B be a closed set and let A=$f^{-1}$(B). Let x$\in$ $cls_\theta$ A. Then f(x)$\in$ f($cls_\theta$A) $\subseteq$cls(f(A))$\subseteq$ cls(B)=B. Therefore, x $\in f^{-1}$(B)=A. Thus $cls_\theta$ A=A.

(b)⇒ (c) . Let V be an open subset of Y and thus Y\V is closed. Let A= $f^{-1}$(Y\V). Then $f^{-1}$(Y\V) = X \ $f^{-1}$(V) is θ - closed and thus $f^{-1}$(V) is θ-open.

(c)⇒ (d). Let x $\in$X and let V be an open set containing f(x). By the hypothesis, it follows that $f^{-1}$(V) is θ -open and thus there exists U an open set containing x such that cls(U) $\subseteq$ $f^{-1}$(V). Thus f(cls(U)) $\subseteq$V, proving that f is strongly continuous.

(d) ⇒ (a). Let f:X→Y be strongly continuous and let x $\in$$cls_\theta$ A. Let V be an open set containing f(x). By strong continuity of f there exists an open set U containing x such that f(cls(U))$\subseteq$ V. Therefore, cls(U) meets A and thus V meets f(A). Hence f(x)$\in$cls(f(A)) as we claim.

The proofs of the following Lemmas are straightforward from the definitions.

*Lemma 1.* Let $f:X \to Y$ be strongly continuous and let $g:Y \to Z$ be continuous. Then gof is strongly continuous.

*Lemma 2.* Let $f:X \to Y$ be closure continuous and let $g:Y \to Z$ be closure continuous. Then go f is closure continuous.

*Lemma 3.* Let $f:X \to Y$ be closure continuous and let $g:Y \to Z$ be strongly continuous. Then gof is strongly continuous.

*Lemma 4.* Let X or Y be regular. Then $f:X \to Y$ is continuous iff f is strongly continuous.

*Remark.* We conclude from Lemmas 1 & 3 that the composite of two strongly continuous functions is strongly continuous.

In [11] it is shown that a function f is weakly continuous iff its graph mapping g is weakly continuous. This is still true for the case of closure continuity as it is shown in the next Theorem but it is not the case for strong continuity as it is shown in Example 2.

*Theorem 2.* Let $f:X \to Y$ be a mapping and let $g:X \to X\times Y$ be the graph mapping of f given by $g(x)=(x,f(x))$ for every point $x\in X$. Then $g:X \to X\times Y$ is closure continuous iff $f:X\to Y$ is closure continuous.

*Proof.* If g is closure continuous. Then it follows from Lemma 2 that f is closure continuous, since the projection map $\pi: X \times Y \to Y$ is continuous and $f=\pi\ og$. Conversely, assume f is closure continuous and Let $x \in X$ and let W be an open set in $X \times Y$ containing $g(x)$. Then there exist an open set $A \subseteq X$ and an open set $V \subseteq Y$ such that $g(x)=(x,f(x)) \in A\times V \subseteq W$. Since f is closure continuous there exists an open set U containing x such that f (cls (U)) $\subseteq$ cls (V). Let K = U$\cap$ A. Then g (cls (K)) $\subseteq$ cls (A) $\times$ cls (V) = cls (A$\times$ V) $\subseteq$ cls (W), proving that g is closure continuous.

*Theorem 3.* Let f:X$\rightarrow$ Y be a mapping and let g:X$\rightarrow$ X $\times$Y be the graph mapping of f given by g(x)=(x,f(x)) for every point x $\in$X. If g:X $\rightarrow$ X $\times$Y is strongly continuous then f:X $\rightarrow$Y is strongly continuous. Moreover, if the graph mapping g of f is strongly continuous then X is regular.

*Proof.* It follows directly from Lemma 1 that f is strongly continuous, since the projection map $\pi$:X $\times$Y$\rightarrow$ Y is continuous and f=$\pi$ og. To prove the regularity of X. Let x $\in$X and let U be an open set containing x. Then U $\cap$Y is an open set containing (x,f(x)). The strong continuity of the graph mapping of f guarantees the existence of an open set W containing x such that g(cls(W))=cls(W) $\times$ f(cls(W)) $\subseteq$ U $\times$Y. Thus x$\in$cls(W) $\subseteq$U, proving that X is regular.

In [9, Corollary to Theorem 6] it is claimed that the converse of Theorem 3 is also true which is not as it is shown in the next example.

*Example 2.* Let X=Y={1,2,3} with topologies $\Gamma_X$ = {$\phi$,{1},{2},{1,2},X}, $\Gamma_Y$ = {$\phi$,{3},Y}; f(x)=3, for all x. Then f is strongly continuous but the graph mapping g of f, where g(x)=(x,f(x)) is not strongly continuous at 1 and 2.

If the domain of f is a regular space then the converse of Theorem 3 is also true.

*Theorem 4.* Let f: X $\rightarrow$Y be a mapping with X a regular space, and let g: X $\rightarrow$ X $\times$ Y be the graph mapping of f given by g(x)=(x, f(x)) for every point x$\in$ X. If f:X$\rightarrow$Y is strongly continuous then g: X$\rightarrow$ X$\times$ Y is strongly continuous.

*Proof.* Assume f is strongly continuous and Let $x \in X$ and let W be an open set in $X\times Y$ containing g(x). Then there exist an open set $A \subseteq X$ and an open set $V \subseteq Y$ such that $g(x)=(x, f(x))\in A \times V \subseteq W$. Since f is strongly continuous, there exists an open set U containing x such that $f(cls(U)) \subseteq V$. By the regularity of X, there exists an open set K containing x such that $cls(K)\subseteq U \cap A$. Therefore, $g(cls(K)) \subseteq A \times V \subseteq W$, proving that g is strongly continuous.

By a closure retraction we mean a closure continuous function $f:X\rightarrow A$, where $A\subseteq X$ and $f|_A$ is the identity function on A. In this case, A is said to be a closure retraction of X

*Theorem 5.* Let $A\subseteq X$ and let $f:X\rightarrow A$ be a closure retraction of X onto A. If X is a completely Hausdorff space, then A is a $\theta$ -closed subset of X.

*Proof.* Suppose not, then there exists a point $x\in cls_\theta A\setminus A$. Since f is a closure retraction we have $f(x) \neq x$. Since X is completely Hausdorff, there exist open sets U and V of x and f(x), respectively, such that $cls(U)\cap cls(V)=\phi$. Now let W be any open set in X containing x. Then $U\cap W$ is an open set containing x and hence $cls(U\cap W)\cap A\neq\phi$, since $x\in cls_\theta A$. Therefore, there exists a point $y\in cls(U\cap W)\cap A$. Since $y\in A$, $f(y)=y\in cls(U)$ and hence $f(y)\notin cls(V)$. This shows that $f(cls(W))$ is not contained in cls(V). This contradicts the hypothesis that f is closure continuous. Thus A is $\theta$-closed as claimed.

Recall that an almost retraction is an almost continuous function $f:X\rightarrow A$, where $A\subseteq X$ and $f|_A$ is the identity function on A. In this case, A is said to be an almost retraction of X.

*Corollary 1.* Let A be an almost retract of a completely Hausdorff space X. Then A is $\theta$-closed.

*Proof.* The proof follows from Theorem 5, since every almost continuous is closure continuous.

*Corollary 2.* Let A be a retract of a completely Hausdorff space X. Then A is $\theta$-closed. By a strong retraction we mean a strongly continuous function $f:X \to A$, where $A \subseteq X$ and $f|_A$ is the identity function on A. In this case, A is said to be a strong retraction of X.

*Theorem 6.* Let $A \subseteq X$ and let $f:X \to A$ be a strong retraction of X onto A. If X is Hausdorff, then A is a $\theta$-closed subset of X.

*Proof.* Suppose not, then there exists a point $x \in cls_\theta A \setminus A$. Since f is a strong retraction we have $f(x) \neq x$. Since X is Hausdorff, there exist open sets U and V of x and f(x) respectively, such that $cls(U) \cap V = \phi$. Now let W be any open set in X containing x. Then $U \cap W$ is an open set containing x and hence $cls(U \cap W) \cap A \neq \phi$, since $x \in cls_\theta A$. Therefore, there exists a point $y \in cls(U \cap W) \cap A$. Since $y \in A$, $f(y) = y \in cls(U)$ and hence $f(y) \notin V$. This shows that f(cls(W)) is not contained in V. This contradicts the hypothesis that f is strongly continuous. Thus A is $\theta$-closed as claimed.

*Theorem 7.* Let $f:X \to Y$ be a closure continuous and injective function. If Y is completely Hausdorff, then X is completely Hausdorff.

*Proof.* For any distinct points $x_1, x_2 \in X$, we have $f(x_1) \neq f(x_2)$, since f is injective. Since Y is a completely Hausdorff, there exist open sets $V_1, V_2$ of $f(x_1)$ and $f(x_2)$, respectively, such that $cls(V_1) \cap cls(V_2) = \phi$. But since f is closure continuous there exist $U_1, U_2$, of $x_1$, $x_2$, respectively, such that $f(cls(U_1)) \subseteq cls(V_1)$, and $f(cls(U_2)) \subseteq cls(V_2)$. Thus $cls(U_1) \cap cls(U_2) = \phi$, proving that X is completely Hausdorff.

*Corollary 3.* Let $A \subseteq X$ and let $f: X \rightarrow A$ be a bijective closure continuous function. If A is completely Hausdorff, then A is a $\theta$-closed subset of X.

*Proof.* Since A is completely Hausdorff, Theorem 7 implies that X is completely Hausdorff. Therefore, Theorem 5 implies that A is $\theta$-closed.

*Theorem 8.* [9, Theorem 4]. Let $f: X \rightarrow Y$ be a strongly continuous and injective function. If Y is a $T_1$-space, then X is Hausdorff.

*Corollary 4.* Let $A \subseteq X$ and let $f: X \rightarrow A$ be a bijective strongly continuous function. If A is a $T_1$-space, then A is a $\theta$ -closed subset of X.

*Proof.* Since A is $T_1$, Theorem 8 implies that X is Hausdorff. Therefore, Theorem 6 leads that A is $\theta$-closed.

The next theorem was given in [11] but the proof depends on a result from [6], one could give an alternative proof similar to the proof of Theorem 10 below.

*Theorem 9.* Let f,g be weakly continuous from a space X into a completely Hausdorff space Y. Then the set $A=\{x \in X: f(x)=g(x)\}$ is a closed set.

*Theorem 10.* Let f,g be closure continuous from a space X into a completely Hausdorff space Y. Then the set $A=\{x \in X: f(x)=g(x)\}$ is a $\theta$-closed set.

*Proof.* We will show that $X \setminus A$ is $\theta$-open. Let $x \in X \setminus A$. Then $f(x) \neq g(x)$. By complete Hausdorffness of Y there exist open sets U and V containing f(x), and g(x), respectively, such that $cls(U) \cap cls(V) = \phi$. By closure continuity of f and g there exist $U_1, U_2$ open nbhds of x such that $f(cls(U_1)) \subseteq cls(U)$ and $g(cls(U_2)) \subseteq cls(V)$. Let $W = U_1 \cap U_2$, then $cls(W) \subseteq X \setminus A$. Thus $X \setminus A$ is $\theta$-open and hence A is $\theta$-closed.

*Theorem 11.* [9, Theorem 2]. Let f,g be strongly continuous from a space X into a Hausdorff space Y. Then the set $A=\{x\in X:f(x)=g(x)\}$ is a θ-closed set.

*Definition.* A subset A of a space X is said to be θ-dense if its θ-closure equals X

The next corollaries are generalizations to a well-known principle of extension of identities.

*Corollary 5.* Let f,g be closure continuous from a space X into a completely Hausdorff space Y. If f,g agree on a θ-dense subset of X. Then f=g everywhere.

*Proof.* Suppose that $A=\{x\in X:f(x)=g(x)\}$ is θ-dense. By Theorem 10, A is θ-closed. Thus A=X.

*Corollary 6.* Let f,g be weakly continuous from a space X into a completely Hausdorff space Y. If f,g agree on a dense subset of X. Then f=g everywhere.

*Proof.* Suppose that $A=\{x\in X:f(x)=g(x)\}$ is dense. By Theorem 9, A is closed. Thus A=X.

*Corollary 7.* Let f,g be strongly continuous from a space X into a Hausdorff space Y. If f,g agree on a θ-dense subset of X. Then f=g everywhere.

*Proof.* Suppose that $A=\{x\in X:f(x)=g(x)\}$ is θ-dense. By Theorem 11, A is θ-closed. Thus A=X.

We conclude this paper with some decomposition theorems of closure continuity and strong continuity. First we need some lemmas from [8], [11], [16].

*Lemma 5.* [11, Theorem 4]. Let $f:X\rightarrow Y$ be a weakly continuous function. Then $cls(f^{-1}(V))\subseteq f^{-1}(cls(V))$, for every open set $V\subseteq Y$.

*Lemma 6.* [8, Lemma to Theorem 4]. Let $f:X \rightarrow Y$ be an open function. Then $f^{-1}(cls(V)) \subseteq cls(f^{-1}(V))$, for every open set $V \subseteq Y$.

*Lemma 7.* [16, Theorem 4]. An open function $f:X \rightarrow Y$ is weakly continuous iff it is a.c.S.

The following results are some decomposition theorems for different forms of continuity which are similar to those in [8] and [11]. The next result is a stronger result of Theorem 5 in [11].

*Theorem 12.* Let $f:X \rightarrow Y$ be a.c.H. and $cls(f^{-1}(V)) \subseteq f^{-1}(cls((V))$ for every open set $V \subseteq Y$. Then f is closure continuous.

*Proof.* Let $x \in X$ and let V be an open nbhd of f(x). Since f is a.c.H and by our hypothesis, $cls(f^{-1}(V))$ is a nbhd of x and thus there exists U an open set in X containing x such that $cls(U) \subseteq cls(f^{-1}(V)) \subseteq f^{-1}(cls(V))$. Therefore $f(cls(U)) \subseteq cls(V)$, proving that f is closure continuous.

*Corollary 8.* An a.c.H. function $f:X \rightarrow Y$ is closure continuous iff $cls(f^{-1}(V)) \subseteq f^{-1}(cls(V))$ for every open set $V \subseteq Y$

*Proof.* Since every closure continuous is weakly continuous, the proof follows directly from Lemma 5 and Theorem 12.

*Corollary 9.* A weakly continuous function which is a.c.H is closure continuous.

*Proof.* The proof follows directly from Lemma 5 and Theorem 12.

*Theorem 13.* An open a.c.H function $f:X \rightarrow Y$ is closure continuous iff $cls(f^{-1}(V)) = f^{-1}(cls(V))$ for every open set $V \subseteq Y$.

*Proof.* Let f be closure continuous. Lemma 7 implies that f is a.c.S. Thus by Corollary to [8, Theorem 7], it follows that $cls(f^{-1}(V))) = f^{-1}(cls(V))$, for every open set $V \subseteq Y$. Conversely, let $x \in X$ and let V be an open nbhd of f(x). Since f is a.c.H., there exists an open set U containing x such that

$cls(U) \subseteq cls(f^{-1}(V)) = f^{-1}(cls(V))$. Thus $f(cls(U)) \subseteq cls(V)$, proving that f is closure continuous.

*Theorem 14.* Let $f:X \to Y$ be an open and weakly continuous. Then f is a.c.H.

*Proof.* Let $x \in X$, and let V be open set containing f(x) in Y. Since f is weakly continuous, there exists an open set U containing x such that $f(U) \subseteq cls(V)$. Thus $U \subseteq f^{-1}(cls(V))$. Since f is open, Lemma 6 implies that $f^{-1}(cls(V)) \subseteq cls(f^{-1}(V))$ and thus $U \subseteq cls(f^{-1}(V))$, proving that f is a.c.H.

*Theorem 15.* Let $f:X \to Y$ be a.c.H. and $cls(f^{-1}(V)) = f^{-1}(V)$ for every open set $V \subseteq Y$. Then f is strongly continuous.

*Proof.* Let $x \in X$ and let V be an open nbhd of f(x). Since f is a.c.H and by our hypothesis, $cls(f^{-1}(V))$ is a nbhd of x and thus there exists U an open set in X containing x such that $cls(U) \subseteq cls(f^{-1}(V)) = f^{-1}(V)$. Therefore $f(cls(U)) \subseteq V$, proving that f is strongly continuous.

Recall that a subset of a topological space is called closure compact if each open cover of the set contains a finite subcollection whose closures cover the set.

In [3], [14] it is shown that the image of compact is closure compact under weakly continuous functions and the image of closure compact is closure compact under closure continuous functions. The next result is similar for strong continuity.

*Theorem 16.* Let $f:X \to Y$ be strongly continuous and let K be a closure compact subset of X. Then f(K) is a compact subset of Y.

*Proof.* Let $\mathcal{V}$ be an open cover of f(K). Then $\mathbf{U} = \{V \in \mathcal{V} : V \cap f(K) \neq \phi\}$ is an open cover of f(K). For each $k \in K$, $f(k) \in V_k$ for some $V_k \in \mathbf{U}$. By strong continuity of f there exists an open set $U_k \subseteq X$ containing k such that $f(cls(U_k)) \subseteq V_k$. The collection $\{U_k : k \in K\}$ is an open cover of K and so since K is closure compact there is a finite subcollection $\{U_k : k \in K_0\}$, where $K_0$ is a finite subset of K and $\{cls(U_k) : k \in K_0\}$ covers K. Clearly $\{V_k : k \in K_0\}$ covers f(K) and thus f(K) is a compact subset of Y.

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