# A STUDY OF A FLEXIBLE SUBMERGED CYLINDRICAL TANK SUBJECT TO LATERAL GROUND EXCITATION

Amin H. Helou[1]

## ملخص

في خزانات الزيت الصلبة المغمورة يمكن حساب الضغوط الهيدروديناميكية الداخلية والخارجية بطرق مستقلة . هذه الضغوط تنتج داخليامن حركة الأمواج في الداخل أما خارجيا فالضغط ينتج عن الامواج المشعه . ولكن عندما يكون جسم الخزان لين فالقوى الداخلية والخارجية تتشابك بواسطة ليونة جسم الخزان .

فيما يلي حل رياضي لحالات الخزانات اللينة وهو عبارة عن امتداد لحلول الخزانات الصلبة .

## ABSTRACT

In a submerged rigid oil storage tank it is possible to evaluate the internal and external hydrodynamic forces separately. Internally the forces are due to internal waves generated at the oil-water interface. Externally, they are due to radiated waves. When such a tank is made of steel, at it is usually the case, the internal and external forces get coupled through the flexibility of the shell, hence their solutions become inseparable.

The following is an analytical treatment for the case of a flexible tank: it expands the solution of the rigid tank to include the shell flexibility effect. The submerged tank is assumed to be subject to a harmonic lateral ground excitation.

[1] Engineering Department, An-Najah N. University, Nablus, West-Bank

# INTRODUCTION

The hydrodynamic forces on the wall of a rigid tank both internally and externally were studied separately by various authors like Fisher[1]. Valestos and Yang[5], Tung[4], and Helou[2][3]. That was possible because the tank's wall was assumed to be infinitely rigid and the fluid motions inside and outside the tank were independent. When the tank is flexible the fluid motions inside and outside as well as their solutions become inseparable. Based on the author's previous work[2][3] gravity effects are neglected in the following solution of the fluid.structure interaction problem. The modes of vibration of circular cylindrical shell are usually defined by the two integers $\bar{n}$ and $\bar{m}$ ; the integer $\bar{n}$ refers to the number of circumferential waves while $\bar{m}$ refers to number of axial waves. Any combination of $\bar{n}$ and $\bar{m}$ defines a natural mode and an associated natural frequency. Figure 1 shows some representative modes of vibration of such shells. For the dynamic analysis of the fluid tanks and tower structures subject to earthquake excitation the modes associated with $\bar{n}$ = 0 and 1 are of interest. It is generally believed that the mode corresponding to $\bar{n}$ = 0 , known as the breathing mode, is excited during vertical ground excitation, irrelevant for this study. For tanks with a rigid top the use of mode corresponding to $\bar{n}$ = 1 is well justified. Oil storage tanks generally have a height to radius ratio in the neighborhood of 1 which make their modelling as a cantilever shear beam quite appropriate. This suggestion is supported by the works of Yang[7], and Wu et.al.[6].

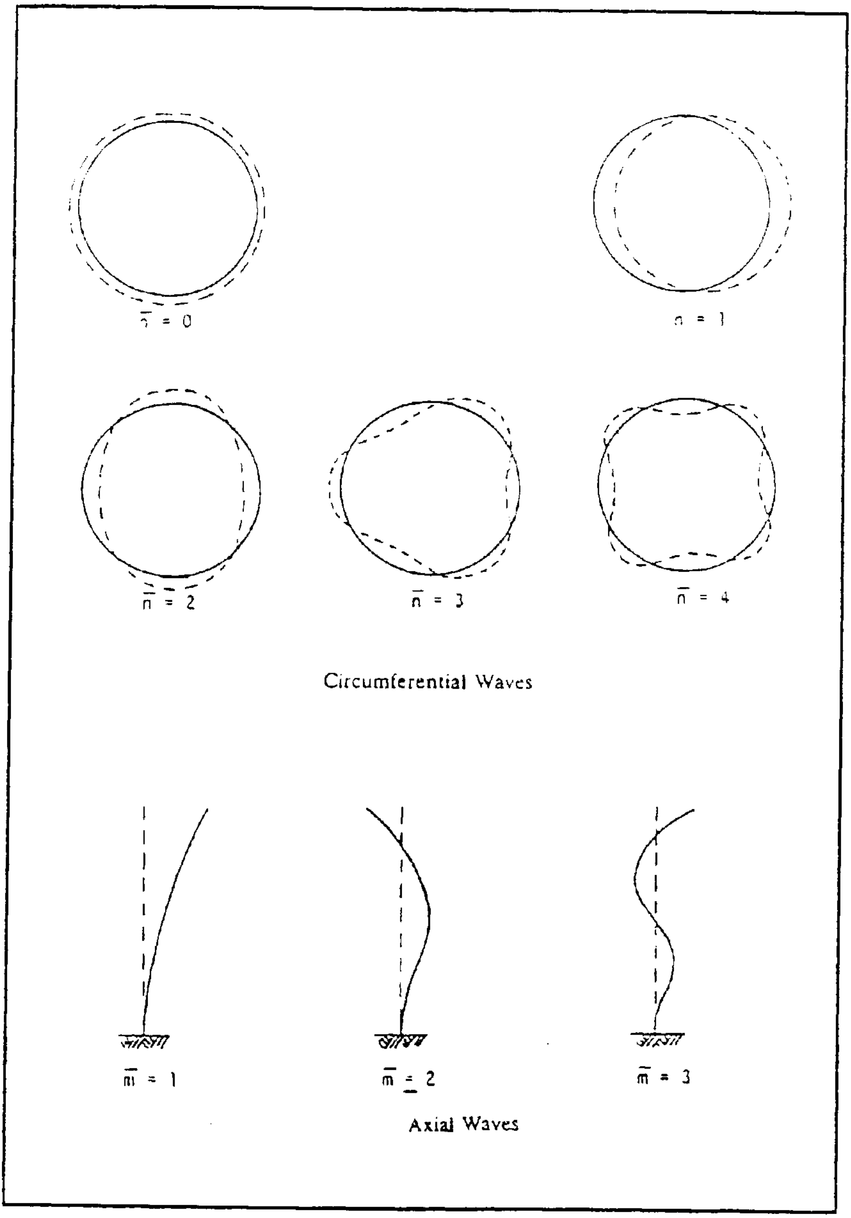


**Figure 1: Modes of vibration of cantilever shells**

Standard methods of mechanics give the cantilever shear mode as follows:

$$X_n = \sin \overline{p_n} x \tag{1}$$

where

$$\bar{P}_n = \frac{(2n-1)\pi}{2L} \qquad n = 1, 2, 3 \ldots \infty \tag{2}$$

and the natural frequency as

$$P_n = \frac{(2n-1)\pi}{2L} \sqrt{\frac{k'G}{\rho}} \qquad n = 1, 2, 3 \ldots \infty \tag{3}$$

in which $k'$ is a factor that depends on the shape of the cross section. G is the shear modulus of elasticity, $\rho$ is the mass per unit length and L is the height of the tank.

Fundamental to the analysis is the assumption that the cross section remains circular during deformation, i.e. $\bar{n} = 1$ and that the deflection configuration is of prescribed form. The first mode corresponding to $\bar{m}$ = 1 is important others are not. This will be substantiated during the course of this presentation.

## Statement of the problem and its solution:

Consider the submerged tank shown in figure 2. Its roof has the-same thickness $h_s$ as the wall; $\rho_s$ is its mass density. Following the assumption that the tank behaves as a beam the total displacement at any point may be written as:

$$V(z,t)=f_H(t)+\psi_1(z)\,Y_1(t) \tag{4}$$

Where $\psi_1(z)$ is a shape function and $Y_1$ is the generalized coordinate amplitude.

Once the flexibility effect is introduced, the Bernoulli's equation for determining the internal pressure on the tank's wall is modified to take the form:

$$P_i=-\rho_i\left\{\frac{\partial\Phi_i}{\partial t}+(\ddot{f}_H(t)+\psi_1(z)\,\ddot{Y}_1(t))\,r\cos\theta+gz\right\} \tag{5}$$

This shows that the flexibility effect is reflected only in the frequency independent component of the pressure. Therefore, the

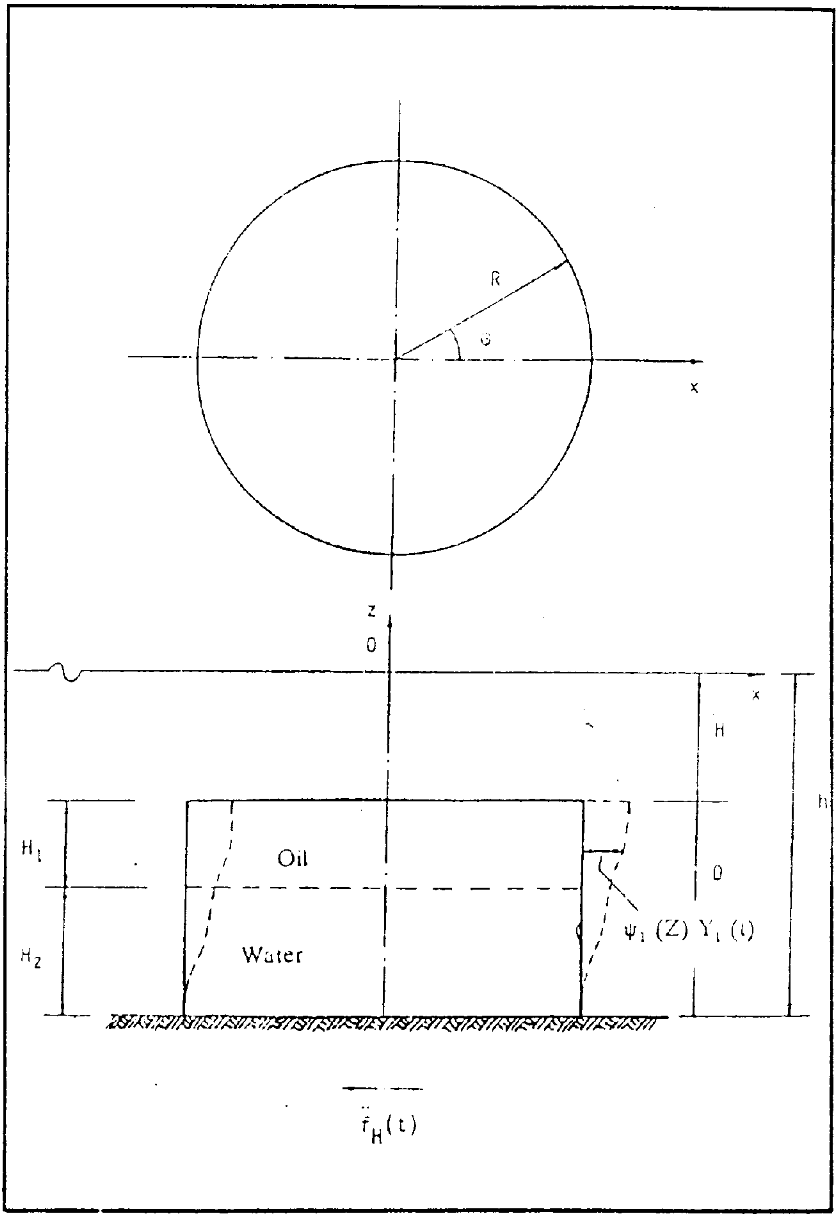


Figure 2: Definition sketch for a submerged circular cylindrical tank vibrating with an assumed mode shape.

internal pressure is conveniently written as:

$$P_{DI}^{i} = P_{I0}^{i} + P_{I1}^{i} \tag{6}$$

in which i = 1 denotes upper layer - oil
= 2 denotes lower layer - water

In a similar manner, the solution to the exterior wave radiation problem is modified to incorporate the extra displacement caused by the flexibility of the wall. Although the details of the solution are given by Helou[3] it must be noted that the assumptions of inviscid and incompressible fluid, irrotational motion and small amplitude waves still hold. The fluid velocity at r = R is of the form standard $V(Z)\cos\theta e^{-i\omega t}$ where V(z) is an unknown function of z found to be:

$$V(z) = \sum_{n=1}^{\infty} b_n f_n(z) \overline{K_n} K_1^{\prime}(\overline{K_n} R) \qquad -H < z < 0 \tag{7}$$

$$V(z) = \sum_{n=1}^{\infty} B_n F_n(z) \overline{k_n^{*}} I_1^{\prime}(\overline{k_n^{*}} R) \qquad -H < z < 0 \tag{8}$$

and at the tanks wall

(9)

$$V(z) = -\frac{1}{i\omega} + \psi_1(z)\,\overline{Y_1}(\omega)\,(i\omega) \qquad -h < z < -H$$

The methodology involved in obtaining the above expression is the same as that for the rigid tank case, i.e. eigenvalue expansion. The expression for $B_n$ is the same as that obtained for a rigid tank[3] but the expression for $b_{nf}$ takes the following modified form:

(10)

$$b_{nf}(t) = b_n + \frac{\alpha_{1n}}{K_1'(\overline{k_n}R)\,\overline{k_n}}\,\dot{Y}_1(t)$$

where $b_n$ is the same value obtained for the perfectly rigid tank[3] and the second term accounts for the flexibility effects

(11)

$$\alpha_{1n} = \int_{-h}^{-H} \psi_1(z)\, f_n(z)\, dz$$

From the above it appears possible to express the external pressure as follows

$$P_{DE} = P_{E0} + P_{E1}$$

where $P_{E0}$ is the hydrodynamic pressure component at the tanks wall due to the rigid body motion and $P_{E1}$ is the correction induced due to the flexibility of the tank.

## The equation of motion of the structure and its solution:

Following the assumption that the tank behaves as a beam, the equation of motion for the beam in its fundamental mode of vibration subjected to a horizontal ground excitation can be written. By equating the work done at any time t by the external forces during a virtual displacement $\delta Y(\omega,t) = \omega_1(z)\delta Y$ to the work done by the internal forces, the following equation of motion is obtained

$$f_I^*\delta Y+f_D^*\delta Y+f_S^*\delta Y+P_E^*\delta Y+P_I^*\delta Y=0 \tag{12}$$

where

$f_I^*$ is the generalized inertia force

$f_D^*$ is the generalized damping force

$f_S^*$ is the generalized elastic force

$P_E^*$ is the generalized exterior hydrodynamic force

$P_I^*$ is the generalized interior hydrodynamic force

$\delta Y$ is a virtual displacement

A generalized quantity is defined as

$$F^*=\int F(z,t)\,\psi_1(z)\,dz \tag{13}$$

The generalized inertia force is easily observed to be

$$f_I^* = A_1 \ddot{f}_H(t) + B_1 \ddot{Y}_1(t) \tag{14}$$

where

$$A_1 = \int_{-h}^{-H} \mu(z)\,\psi_1(z)\,dz + m_r \psi_1(D) \tag{15}$$

in which $m_r$ is the mass of the tank's roof and

$$B_1 = \int_{-h}^{-H} \mu(z)\,\psi_1^2(z)\,dz + m_r \psi_1^2(D) \tag{16}$$

Similarly the generalized exterior hydrodynamic force may be written as

$$P_E^* = A_2 \ddot{f}_H(t) + B_2 \ddot{Y}_1(t) \tag{17}$$

where

$$A_2 = \pi \int_{-h}^{-H} P_{EO}(z)\,\psi_1(z)\,R\,dz \tag{18}$$

and

$$B_2 = \pi \int_{-h}^{-H} P_{EI}(z)\,\psi_1(z)\,R\,dz \tag{19}$$

Finally, the generalized interior hydrodynamic pressure is written as:

$$P_I^* = A_2 \ddot{f}_H(t) + B_3 \ddot{Y}_1(t) \tag{20}$$

where

$$A_3 = \pi \int_0^{H_1} P_{I0}^1(z)\,\psi_1(z)\,R\,dz + \pi \int_{-H_2}^{0} P_{I0}^2(z)\,\psi_1(z)\,R\,dz \tag{21}$$

and

$$B_3 = \pi \int_0^{H_1} P_{I1}^1(z)\,\psi_1(z)\,R\,dz + \pi \int_{-H_2}^{0} P_{I1}^2(z)\,\psi_1(z)\,R\,dz \tag{22}$$

The equation of motion (12) may now be written in the following form:

$$\ddot{Y}_1 + 2\xi^*\omega_1^*\dot{Y}_1(t) + \omega_1^{*2} Y_1(t) = -\bar{c}\ddot{f}_H(t) \tag{23}$$

where

$$\bar{C}=\frac{A_1+A_2+A_3}{B_1+B_2+B_3} \tag{24}$$

and

$$\omega_1^*=\omega_1\sqrt{\frac{B_1}{B_1+B_2+B_3}} \tag{25}$$

$$\xi^*=\xi\sqrt{\frac{B_1}{B_1+B_2+B_3}} \tag{26}$$

The quantities $\omega_1$ and $\omega^*_1$ refer to the natural frequency of oscillation of the structure in air and in water respectively. $\xi$ and $\xi^*$ are the respective modal damping ratio of the structure.

Upon examination of the previous equations it becomes clear that the hydrodynamic pressures at the wall inside and outside the tank have effects equivalent to "added masses" $B_2$ and $B_3$ and "added excitations" $A_2$ and $A_3$. It is further realized from equation (25) and (26) that resulting from the fluid-structure interaction of the structure the natural frequency of oscillation of the structure as well as its modal damping ratio are decreased proportionately to the added masses $B_2$ and $B_3$.

Since the right-hand side of equation (23) is proportional to $e^{-i\omega t}$ the response can be assumed to be the form

(27)

$$Y_1(t) = \overline{Y_1}(\omega)\, e^{-i\omega t}$$

which gives the solution for $\ddot{Y}_1(t)$ to be

(28)

$$\ddot{Y}(t) = \frac{\overline{C}}{\left(1 - \frac{\omega_1^{*2}}{\omega_2}\right) + 2i\xi^* \frac{\omega_1^*}{\omega}} \cdot e^{-i\omega t}$$

or equivalently

(29)

$$\ddot{Y}_1 = -\overline{B}\left[\left(1 - \frac{\omega_1^{*2}}{\omega^2}\right)\cos\omega t - 2\xi^* \frac{\omega_1^*}{\omega}\sin\omega t - i2\xi^* \frac{\omega_1^*}{\omega}\cos\omega t - i\left(1 - \frac{\omega_1^{*2}}{\omega^2}\right)\sin\omega t\right]$$

where

(30)

$$\overline{B} = -\frac{\overline{C}}{\left(1 - \frac{\omega_1^{*2}}{\omega^2}\right) + 2i\xi^* \frac{\omega_1^*}{\omega}}$$

From the above equation a value for $\dot{Y}(t)$ is readily obtained. Accordingly, the velocity potentials in the upper and lower layers inside the tank as well as in region 1 surrounding the tank become fully determined. Extracting the real part of the solution only the pressures are written as

(31)

$$P_{DI}^{1}=-\rho_1 R\cos\theta\{\cos\omega t-\psi_1(z)\bar{B}[(1-\frac{\omega_1^{*2}}{\omega^2})\cos\omega t-2\xi^{*}\frac{\omega_1^{*}}{\omega_1}\sin\omega t]\}$$

and

(32)

$$P_{DI}^{2}=-\rho_2 R\cos\theta\{\cos\omega t-\psi_1(z)\bar{B}[(1-\frac{\omega_1^{*2}}{\omega^2})\cos\omega t-2\xi^{*}\frac{\omega_1^{*}}{\omega_1}\sin\omega t]\}$$

From which the amplitude of the hydrodynamic pressure taken at $\theta = 0$ for the upper layer is computed as

(33)

$$P_{DI}^{1}=-\rho_1\sqrt{P_S^2+P_C^2}$$

and for the lower layer

(34)

$$P_{DI}^{2}=-\rho_2\sqrt{P_S^2+P_C^2}$$

where the terms $P_s$ and $P_c$ are defined as

(35)

$$P_S = 2R\xi^{*}\frac{\omega_1^{*}}{\omega}\psi(z)\overline{B}$$

and

(36)

$$P_C = R\left[1-\psi_1(z)\overline{B}\left(1-\frac{\omega_1^{*2}}{\omega^2}\right)\right]$$

In a similar manner, the pressure distribution in region 1 surrounding the tank can be express as

(37)

$$P_{1f} = \rho_1\sqrt{P_S^{2\prime}+P_C^{2\prime}}$$

where

(38)

$$P_S^{\prime} = \sum_{n=1}^{\infty} b_{nr} - \frac{a_{1n}}{K_1^{\prime}(\overline{K}_n R)\overline{K}_n}\overline{B}\left(1-\frac{\omega_1^{*}}{\omega^2}\right) f_n(z)\, k_1(\overline{K}_n R)\ \cos\theta$$

$$P_C' = \sum_{n=1}^{\infty} \frac{2\alpha_{1n}}{K_1'(\overline{k}_n R)\,\overline{k}_n} \overline{B}\xi^{*} \frac{\omega_1^{*}}{\omega} f_n(z) K_1(k_n R) \cos\theta \tag{39}$$

in which $b_{nr} = b_n e^{i\omega t}$

## RESULTS AND CONCLUSION

Figure 3 shows the interior and exterior hydrodynamic pressure distributions taken at $\theta = 0$ for a tank of radius equal to 10 meters and a height equal to 10 meters. The tank has a rigid roof and wall thickness equal to 2.5 centimeters and submerged in water 20 meters deep. It is made of steel with a relative density equal to 7.85 and assumed to be subject to 5% structural damping. The results shown in figure 3 are based on the assumption that the tank behaves as a shear beam and vibrates in its first mode only. As a result of the added mass, the frequency of oscillation of the tank is reduced from 340 rad/sec to 80 rad/sec. Similarly, the structural damping is reduced from 5% to 1% in water.

For an empty tank made of steel with a height equal to 20 meters the natural frequency of oscillation in air corresponding to the first mode of vibration in shear deformation is 170 radians per second while that corresponding to the second mode is three times as much. since the height of submerged tanks would not conceivably be higher than 20 meters and since structures with natural frequencies greater than 200 radians per second are, for all practical purposes, considered rigid, it is therefore concluded that the first mode of vibration is predominant and that the response due to higher modes can safely be neglected. This further justifies and reinforces the assumptions made of the onset of this study.

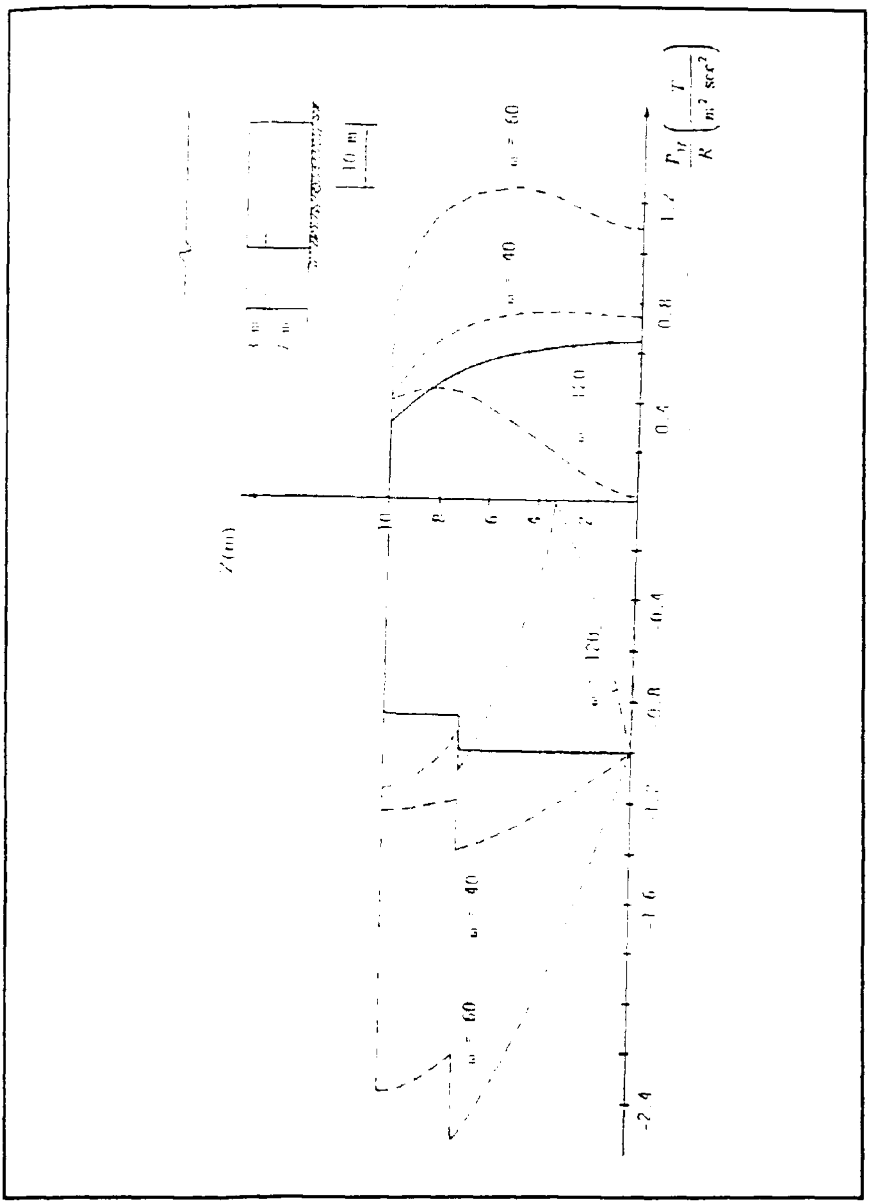

Figure 3 : Interior and exterior hydrodynamic pressure distribution at the wall of a circular cylindrical tank taken at 0 = 0 (--- right case, ----- w dependent)

# REFERENCES


1. Fischer, D., "Dynamic Fluid Effects on Liquid-Filled Flexible Cylindrical Tanks", Earthquake Engineering and Structural Dynamics, Vol. 7, 1979.
2. Helou, A.H., "Dynamics of Internal Waves in Cylindrical Tank," "Journal of Waterway Port, Coastal and Ocean Engineering Vol. 111 No. 2, March 1985.
3. Helou, A.H. "A Detailed and Simplified Solution to Hydrodynamic Forces on a Submerged Tank Subject to Lateral Ground Excitation, "An-Najah Journal of Research, Vol. 1, Section 2, Jan. 1988.
4. Tung, C.C., "Hydrodynamic Forces of Submerged Vertical Circular Tanks Under Ground Excitation, "Applied Ocean Engineering, Vol. 1, No. 2, April 1979.
5. Valestos, A. S., and Yang, J.Y., "Dynamics of Fixed-Base Liquid Storage Tanks," Proceedings of U.S. -- Japan Seminar for Earthquake Engineering Research with Emphasis on Life Line Systems, Tokyo, Japan, November 1976.
6. Wu, C.I., Mouzakis, T.K., Nash, W.A. and Colonell, J.M., "Natural Frequencies Cylindrical Liquid Storage Containers",Engineering Research Institute University of Massachusetts, 1975.
7. Yang, S.Y., "Dynamic Behavior of Fluid-Tank systems", Ph.D. Dissertation, Rice University, Houston, Texas, March 1976.